

$$1.) f'(x) = 3x^2 + 10x - 8 = (3x - 2)(x + 4)$$

$$f'(x) = 0 \text{ when } x = -4 \text{ and } x = \frac{2}{3}$$

$$f''(x) = 6x + 10 \quad f''(-4) = -14 < 0 \quad f''\left(\frac{2}{3}\right) = 14 > 0$$

$f(x)$  has a local max at  $x = -4$  b/c  $f'(-4) = 0$   
and  $f''(-4) < 0$ .

$$2.) f'(x) = \sqrt{3} - 2\cos x \rightarrow f'(\pi/6) = 0$$

$$f''(x) = 2\sin x \rightarrow f''(\pi/6) = 1 > 0$$

$f(x)$  has a local min at  $x = \pi/6$  b/c  $f'(\pi/6) = 0$   
and  $f''(\pi/6) > 0$

$$3.) f'(x) = 3x^2 - 6x = 3x(x - 2) = 0 \text{ when } x = 0, 2$$

$$f''(x) = 6x - 6 \rightarrow f''(0) = -6 < 0 \text{ so } f(x) \text{ has a local max at } x = 0$$

$$f''(2) = 6 > 0 \text{ so } f(x) \text{ has a local min at } x = 2.$$

$$4.) f'(x) = 1 - \frac{4}{x^2} = 0 \text{ when } x = \pm 2$$

$$f''(x) = \frac{8}{x^3} \rightarrow f''(2) = 1 > 0 \text{ so } f(x) \text{ has a local min at } x = 2$$

$$f''(-2) = -1 < 0 \text{ so } f(x) \text{ has a local max at } x = -2.$$

$$5.) f'(x) = \cos x + \sin x = 0 \text{ when } x = 3\pi/4, 7\pi/4.$$

$$f''(x) = -\sin x + \cos x$$

$$f''(3\pi/4) = -\frac{2}{\sqrt{2}} < 0 \text{ so } f(x) \text{ has a local max at } x = \frac{3\pi}{4}$$

$$f''(7\pi/4) = \frac{2}{\sqrt{2}} > 0 \text{ so } f(x) \text{ has a local min at } x = \frac{7\pi}{4}$$

$$6) a) \partial_x + 8y y' = 3xy' + 3y \rightarrow 8yy' - 3xy' = 3y - 2x$$

$$y'(8y - 3x) = 3y - 2x \rightarrow y' = \frac{3y - 2x}{8y - 3x}$$

$$b) 3^2 + 4y^2 = 7 + 3(3)y \rightarrow 2 + 4y^2 - 9y = 0$$

$$(4y - 1)(y - 2) = 0 \text{ so } y = 1/4 \text{ or } y = 2.$$

Since the tangent line is horizontal  $y' = 0$   
 $3y - 2x = 0$  at  $(3, 2)$  but not at  $(3, 1/4)$

$P(3, 2)$

$$c.) \frac{d^2y}{dx^2} = \frac{(8y - 3x)(3y' - 2) - (3y - 2x)(8y' - 3)}{(8y - 3x)^2}$$

Note  $y' = 0$   
at  $(3, 2)$

$$\frac{d^2y}{dx^2} = \frac{(8 \cdot 2 - 3 \cdot 3)(-2) - (3 \cdot 2 - 2 \cdot 3)(-3)}{(8 \cdot 2 - 3 \cdot 3)^2}$$

$$= \frac{-14 + 0}{49} = -\frac{14}{49}$$

The curve has a local max at  $(3, 2)$

b/c  $\frac{dy}{dx} = 0$  and  $\frac{d^2y}{dx^2} < 0$ .

7.) a)  $f$  is inc. on  $(-\infty, 0) \cup (3, \infty)$  b/c  $f' > 0$   
 $f$  is dec. on  $(0, 3)$  b/c  $f' < 0$

b.)  $f$  has a local max at  $0$  b/c  $f'$  changes from  $+$  to  $-$ .  
 $f$  has a local min at  $3$  b/c  $f'$  changes from  $-$  to  $+$ .

8.) a)  $f$  is inc. on  $(-1, 3) \cup (5, \infty)$  b/c  $f' > 0$   
 $f$  is dec. on  $(-\infty, -1) \cup (3, 5)$  b/c  $f' < 0$

b.)  $f$  has a local max at  $x=3$  b/c  $f'$  changes from  $+$  to  $-$ .  
 $f$  has a local min at  $x=-1$  and  $x=5$   
b/c  $f'$  changes from  $-$  to  $+$ .

9.)  $f$  has an inflection point at  $x=2$  b/c  $f'$  changes and  $x=6$  from inc. to dec.  
 $f$  has an inflection point at  $x=4$  b/c  $f'$  changes from dec. to inc.

10.)  $f'(x) = 3x^2 + 2ax + b$   
 $f''(x) = 6x + 2a$

$f'(-3)$  must  $= 0$   
 $f'(-1)$  must  $= 0$   
 $f''(-3)$  must be  $< 0$   
 $f''(-1)$  must be  $> 0$

$b = 6a - 27 \rightarrow 3 - 2a + 6a - 27 = 0 \rightarrow 4a = 24 \rightarrow a = 6$

$f''(-3) = -18 + 12 < 0 \checkmark$

$b = 9$

$f''(-1) = -6 + 12 > 0 \checkmark$

11.)		f	f'	f''
	A	+	+	-
	B	+	0	-
	C	+	-	+

$$12.)^a) h(2) = f(g(2)) = f(1) \approx 3.25$$

$$b.) h'(x) = f'(g(x)) \cdot g'(x) \rightarrow h'(1) = f'(g(1)) \cdot g'(1)$$

$$h'(1) \approx f'(1.9) \cdot g'(1) \approx -\frac{1}{4} \cdot (-1) \approx \frac{1}{4}$$

$$c.) h'(3) = f'(g(3)) \cdot g'(3) = f'(3) \cdot g'(3) = (-)(+) = -$$

h is dec. at  $x=3$  b/c  $h'(3) < 0$

$$d.) h'(x) = 0 \quad h'(x) = f'(g(x)) \cdot g'(x)$$

$$\text{so } f'(g(x)) = 0 \quad \text{or } g'(x) = 0$$

$$f'(x) = 0 \text{ when } x = 4 \rightarrow g(x) = 4 \text{ when } x = \frac{1}{4} \text{ and } x = 4$$

$$g'(x) = 0 \text{ when } x = 2.$$