

$$1.) T(x) = f(0) + f'(0)(x-0)$$

$$T(x) = -4 + f'(0)x \text{ and since } T\left(\frac{1}{2}\right) = -3$$

$$-3 = -4 + f'(0)\left(\frac{1}{2}\right) \rightarrow 1 = f'(0)\left(\frac{1}{2}\right) \rightarrow f'(0) = 2$$

$$2.) |\text{error}| \leq \frac{40}{2!} \left(\frac{1}{4} - 0\right)^2 = \frac{5}{4}$$

$$6 - \frac{5}{4} \leq f\left(\frac{1}{4}\right) \leq 6 + \frac{5}{4} \rightarrow \frac{19}{4} \leq f\left(\frac{1}{4}\right) \leq \frac{29}{4}$$

$$3.) a.) g(1) = \int_1^1 f(t) dt = 0 \quad g'(x) = f(x) \rightarrow g'(1) = 2$$

$$y = 2(x-1) \rightarrow g(0.5) \approx 2(-.5) = -1$$

$$b.) |\text{error}| \leq \frac{\text{max value of } |g''(x)| \text{ on } [0.5, 1]}{2!} (0.5-1)^2 = \frac{4}{2} \left(\frac{1}{4}\right) = \frac{1}{2}$$

$$|T(0.5) - g(0.5)| \leq \frac{1}{2}$$

$$4.) a.) y - 80 = 128(x-2)$$

$$h(1.9) \approx 80 + 128\left(-\frac{1}{10}\right) = 67.2$$

$67.2 < h(1.9)$  since  $h'(x)$  is increasing on  $[1, 3]$ ,  
 $h(x)$  is concave up on  $(1, 3)$ .

$$b.) |\text{error}| \leq \frac{488}{3} \frac{(1.9-2)^2}{2!} = \frac{488}{6} \left(\frac{1}{100}\right) = \frac{488}{600} < \frac{5}{6}$$