2.2 Basic Differentiation Rules, Day 1

Power Rule: $\frac{d}{dx}\left[x^n\right]=$

Ex. (a) $f(x)=x^3$ (b) $y=\sqrt{x}$ (c) $g(t)=\frac{1}{t^4}$

$f'(x)=$ $y'=$ $g'(t)=$

(d) $f(x)=5$ (e) $\frac{d}{dx}\left[5x^3\right]=$

$f'(x)=$

Derivative of a Constant Rule: If c is a constant, then $\frac{d}{dx}[c]=$

Sums and Differences: $\frac{d}{dx}\left[f(x)+g(x)\right]=$

$\frac{d}{dx}\left[f(x)-g(x)\right]=$

Ex. $f(x)=4x^3-5x^2+7x+3$

$f'(x)=$

Ex. $y=\frac{5x^5-3x^4+4}{x}$

$y'=$

Ex. Determine the point(s) at which the given function has a horizontal tangent line.

$f(x)=x^3-12x$

$\frac{d}{dx}[\sin x] =$	$\frac{d}{dx}[\cos x] =$

Ex. $y = 6x^2 + 5\cos x$

$y' =$

Average Rate of Change of f on [a, b] $= \dfrac{f(b) - f(a)}{b - a}$

(Average rate of change is also called average velocity.)

Instantaneous Rate of Change of f at $x = c$ is $f'(c)$.

(Instantaneous rate of change is also called instantaneous velocity.)

Ex. $f(x) = x^3 - 3x^2 + 4$

(a) Find the average rate of change of f on [1, 5].

(b) Find the instantaneous rate of change of f at $x = 3$.

Ex. Find k so that the function $f(x) = x^2 + kx$ will be tangent to the line $y = 2x - 9$.

Homework: P.115: 1, 9 – 29 eoo, 34, 35, 37 - 53 eoo, 59,65

Note: eoo means every other odd.

9 – 29 eoo would be 9, 13, 17, 21, 25, 29

2.2 Basic Differentiation Rules, Day 2

Ex. Sketch the graph of a function f such that $f'(x) < 0$ for all x and the rate of change of the function is increasing.

Ex. Find a and b so that f is differentiable everywhere.

$$f(x) = \begin{cases} ax^2 + 1, & x \le 2 \\ bx - 3, & x > 2 \end{cases}$$

Ex. At time $t = 0$, a diver jumps from a diving board that is 32 ft above the water with an initial velocity of 16 ft/sec.

Use the position function $s(t) = -16t^2 + v_0 t + s_0$, where $v_0 =$ initial velocity and $s_0 =$ initial position.

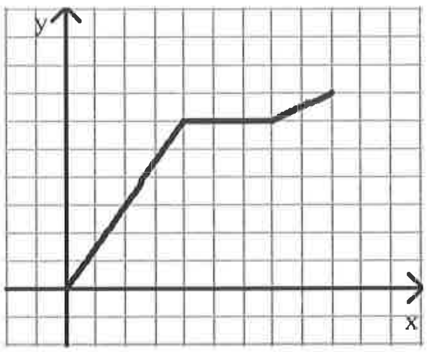
(a) When does the diver hit the water?

(b) Find the instantaneous velocity when $t = 1$ second.

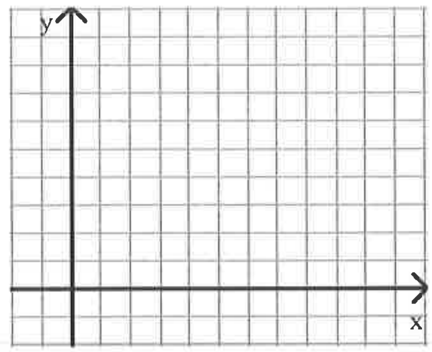
(c) Find the average velocity on the interval [1, 2].

$$\text{Average velocity on } [a, b] = \frac{s(b) - s(a)}{b - a}$$

Ex. The graph of a position function is shown. Sketch the velocity function on (0, 9).



Position function



Velocity function

Homework: P. 117: 83-88, 90, 93, 97,98, 113, 114

2.3 Differentiation Rules for Products, Quotients, Secants, and Tangents

Product Rule: $d\left[f(x)\cdot g(x)\right]=$

Ex. $y=x^3\cos x-2\sin x$. Find y'.

Quotient Rule: $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]=$

Ex. $y=\dfrac{\sin x}{x^3}$

$y'=$

Ex. $f(x)=\dfrac{x^2+2}{\sqrt[4]{x}}$

$f'(x)=$

If you have a quotient in which the numerator is a constant, it's easier to rewrite it as a function with a negative exponent and use the Power Rule.

Ex. $y=\dfrac{9}{5x^2}$

$y'=$

We can use the quotient rule to derive a rule for the derivative of the other trig functions.

$\dfrac{d}{dx}[\tan x]=$

$\frac{d}{dx}[\sec x] =$

The derivative of cot x and csc x can be derived in the same way.

$\frac{d}{dx}[\tan x] =$	$\frac{d}{dx}[\cot x] =$
$\frac{d}{dx}[\sec x] =$	$\frac{d}{dx}[\csc x] =$

<u>Ex</u>. $f(x) = 3x^2 \tan x$

$f'(x) =$

Sometimes we are given the values of a function and its derivative to help us evaluate a product or quotient.

<u>Ex</u>. $g(2) = 3,\ g'(2) = -4,\ h(2) = -1,\ h'(2) = 5.$

$f(x) = g(x)h(x)$. Find $f'(2)$.

<u>Homework:</u> P. 126: 5 – 11 odd, 17, 18, 29, 37 – 53 eoo, 61, 71, 75, 81, 82
"eoo" means every other odd so 37 – 53 eoo means 37, 41, 45, 49, 53

2.4 Derivative of a Composite Function – The Chain Rule

In Algebra, you learned how to find a composition of two functions $f(x)$ and $g(x)$, which was written symbolically as $f(g(x))$.

If $f(x)=x^7$ and $g(x)=3x^2+5$, then $f(g(x))=(3x^2+5)^7$.

If $f(x)=\sin x$ and $g(x)=x^3$, then $f(g(x))=\sin(x^3)$.

If $f(x)=x^3$ and $g(x)=\cos x$, then $f(g(x))=(\cos x)^3$, which we write as $f(g(x))=\cos^3 x$.

"Plain" Functions	Composite Functions
$y=5x^7$	$y=(3x^2+5)^7$
$f(x)=\sin x$	$f(x)=\sin(4x)$
$y=\cos x$	$y=\cos^3 x$

In Calculus, when we need to find the derivative of a composition of functions, we use the Chain Rule.

In problems such as $y=(3x^2+5)^7$, which you learned to differentiate last year, the Chain Rule can be written this way:

Chain Rule: If $y=(u)^n$, then $y'=n(u)^{n-1}\dfrac{du}{dx}$.

The $\dfrac{du}{dx}$ means that you are multiplying by the derivative of the inside quantity.

Our Calculus book writes the Chain Rule the following way:

Chain Rule: $\dfrac{d}{dx}\left[f(g(x))\right]=$

Ex. Find the derivative. Do not leave negative exponents or complex fractions in your answers.

(a) $f(x)=(4x^3+3x-2)^5$

$f'(x)=$

(b) $y=(x^4+2)^{2/3}$

$y'=$

(c) $y=\dfrac{-7}{(2x-3)^2}$

$y'=$

(d) $g(x)=\dfrac{5x}{\sqrt[3]{x^2+2}}$

$g'(x)=$

We will now use the trig formulas the way they are written on your Formula Sheet:

$\dfrac{d}{dx}[\sin u]=\cos u\,\dfrac{du}{dx}$	$\dfrac{d}{dx}[\cos u]=-\sin u\,\dfrac{du}{dx}$
$\dfrac{d}{dx}[\tan u]=\sec^2 u\,\dfrac{du}{dx}$	$\dfrac{d}{dx}[\cot u]=-\csc^2 u\,\dfrac{du}{dx}$
$\dfrac{d}{dx}[\sec u]=\sec u\tan u\,\dfrac{du}{dx}$	$\dfrac{d}{dx}[\csc u]=-\csc u\cot u\,\dfrac{du}{dx}$

With trig functions such as (e), (f), and (g), the Chain Rule means that your derivative will be found by:

(Derivative of trig function)(Derivative of Angle)

(e) $y=\cos(3x)$

$y'=$

(f) $f(x)=\sin\left(5x^2\right)$

$f'(x)=$

(g) $y=\csc\left(7x^5\right)$

$y'=$

With trig functions such as (h) and (i) where the trig function is raised to a power, the Chain Rule means that your derivative will be found by:

(Power Rule)(Derivative of trig function)(Derivative of Angle)

(h) $y=\cos^4(5x)$

$y'=$

(i) $f(x)=\sec^2\left(4x^3+5\right)$

$f'(x)=$

(j) $y=\tan(\sin x)$

$y'=$

(k) $f(3)=2,\ g(3)=4,\ f(4)=-6,\ f'(3)=-7,\ g'(3)=-5,\ f'(4)=8.$

$h(x)=f(g(x))$. Find $h'(3)$.

Homework: P. 137: 9 – 29 eoo, 45, 47, 49, 55 – 59 odd, 63, 67, 71, 75, 77

2.4 Chain Rule, Day 2

Ex. Find the second derivative of $f(x)=4(x^2-5)^3$.

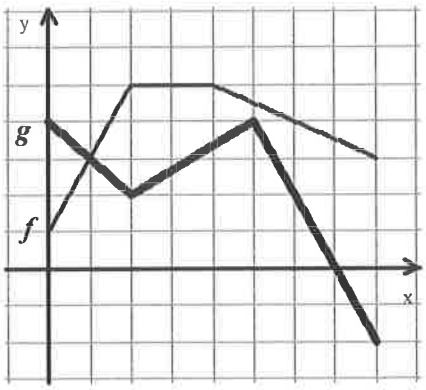
Check your answer on your TI-89 by entering $d\left(4(x^2-5)^3, x, 2\right)$. The 2 tells the calculator to find the second derivative.

Ex. Find the point(s) where f has a horizontal tangent.

$$f(x)=\frac{x}{\sqrt{2x-1}}$$

Ex. Find $g'(x)$ given $g(x)=f(2x^3)$.

Ex.



Use the graphs of f and g to find the following, if they exist.

(a) $h(x) = f(x) \cdot g(x)$. Find $h'(1)$.

(b) $k(x) = \dfrac{f(x)}{g(x)}$. Find $k'(6)$.

(c) $p(x) = f(g(x))$. Find $p'(6)$.

(d) $t(x) = g(f(x))$. Find $t'(7)$.

Homework: P. 138: 87 - 97 odd, 98, 105, 110, 120, 122

2.5 Implicit Differentiation

Examples of **explicitly** defined functions:

$y = x^2 - 5x + 7$

$f(x) = \sqrt{x^2 + 5}$

Examples of **implicitly** defined functions:

$x^2 + xy - y^2 = 3$

$\cos x \sin y = \frac{\sqrt{3}}{4}$

$y^3 + y^2 - x^2 = -4$

Ex. Given $y^3 + y^2 - x^2 = -4$,

(a) Differentiate with respect to t. Since you must apply the Chain Rule, each derivative will have a $\frac{d\_\_}{dt}$ as part of the derivative.

$y^3 + y^2 - x^2 = -4$

(b) Differentiate with respect to w. Since you must apply the Chain Rule, each derivative will have a $\frac{d\_\_}{dw}$ as part of the derivative.

$y^3 + y^2 - x^2 = -4$

(c) Differentiate with respect to x. Since you must apply the Chain Rule, each derivative will have a $\frac{d\_\_}{dx}$ as part of the derivative.

$y^3 + y^2 - x^2 = -4$

Now use Algebra to rearrange and solve for $\frac{dy}{dx}$.

Ex. Find the derivative, $\frac{dy}{dx}$.

(a) $x^2 + xy - y^2 = 3$

(b) $\cos x \sin y = \frac{\sqrt{3}}{4}$

If you have a Titanium TI-89, it has an implicit differentiation command built in. It's under F3.
To check example (a), type: $\text{impdiff}\left(x^2 + x \cdot y - y^2 = 3, x, y\right)$
Since TI-83's and 84's cannot do this, implicit differentiation is always on the non-calculator portion of the AP Test.

Ex. Find the second derivative, $\frac{d^2y}{dx^2}$, given $y^2 + xy = 25$.

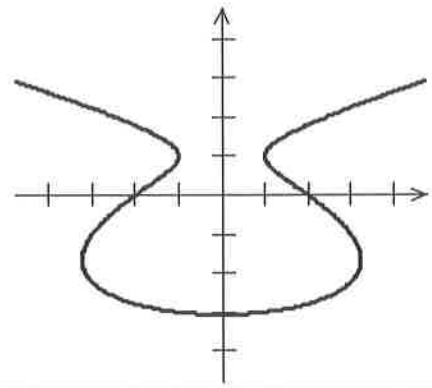
To check this example on your calculator, type: $\text{impdiff}\left(y^2 + x \cdot y = 25, x, y, 2\right)$
The 2 at the end of the command tells the calculator to find the second derivative.

Homework: P. 146: 1 – 33 eoo, 45 – 49 odd,77

2.5 Implicit Differentiation, Day 2

Ex. Consider the curve given by $y^3 + y^2 - 5y - x^2 = -4$.

(a) Find $\frac{dy}{dx}$.



(b) Write the equation of the tangent line to the curve at the point $(1, -3)$.

(c) Find the coordinates of the point(s) on the curve where the line tangent to the curve is vertical.

Homework: Worksheet