

$$1.) S \approx \frac{1}{1} - \frac{1}{8} + \frac{1}{27} - \frac{1}{64} + \frac{1}{125} = .904412$$

1.)  $\lim_{n \rightarrow \infty} \frac{1}{n^3} = 0$  2) Terms are dec. in magnitude

3.) Terms are alternating in sign

|error| < 1<sup>st</sup> omitted term =  $\frac{1}{216}$

$$.899782 < \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} < .909042$$

$$4.) S \approx \frac{1}{1} + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} = 1.18566$$

$$\text{error} < \int_5^{\infty} \frac{1}{x^3} dx = \lim_{b \rightarrow \infty} \int_5^b x^{-3} dx = \lim_{b \rightarrow \infty} \left[ \frac{-1}{2x^2} \right]_5^b \\ = \lim_{b \rightarrow \infty} \left[ \frac{-1}{2b^2} - \frac{-1}{50} \right] = \frac{1}{50}$$

$$1.18566 < \sum_{n=1}^{\infty} \frac{1}{n^3} < 1.20566$$

5.)  $\lim_{n \rightarrow \infty} \frac{3n+1}{5n+2} = \frac{3}{5}$ . The sequence converges to  $\frac{3}{5}$ .

6.)  $\frac{(2n+1)(2n)(2n-1)}{(2n-1)!} \rightarrow \lim_{n \rightarrow \infty} 4n^3 + 2n = \infty$   
The sequence diverges.

7.)  $p = 3/2 > 1 \rightarrow$  series converges

8.) Converges by direct comparison to  $\sum \frac{1}{n^3}$

9.) Converges by ratio test

10.)  $S \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n |r| = \frac{1}{4} < 1$  (convergent geometric series.)

11.)  $\lim_{n \rightarrow \infty} \frac{\frac{n^2}{2}}{2n+1} = \frac{1}{2} \neq 0$  Diverges by  $n^{th}$  term test

12.)  $|r| = \frac{e}{\pi} < 1$  Convergent geometric series.

13.)  $\lim_{b \rightarrow \infty} \int_1^b e^{-x^2} x dx = \left[ \frac{-x^3}{3} \right]_1^b, \frac{du}{dx} = -2x \Rightarrow -\frac{1}{2} du = x dx$

$$\lim_{b \rightarrow \infty} -\frac{1}{2} \left[ e^u \right]_1^b = \lim_{b \rightarrow \infty} -\frac{1}{2} \left[ e^{-x^2} \right]_1^b,$$

$$\lim_{b \rightarrow \infty} -\frac{1}{2} \left[ \frac{1}{e^{b^2}} - \frac{1}{e} \right] = \frac{1}{2e} \text{ which is finite.}$$

The integral converges so the series converges.

14.) ①  $\lim_{n \rightarrow \infty} \frac{3}{4n+1} = 0 \quad -\frac{3}{5} + \frac{3}{9} - \frac{3}{13} + \dots$

② Terms are dec. in magnitude

③ Terms are alt. in sign.

Series converges by A.S.T.

Series diverges by L.C.T.

15.) Compare to  $\sum \frac{1}{n}$

$$16.) \frac{1}{3} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{7}} - \cancel{\frac{1}{9}} + \cancel{\frac{1}{9}} - \cancel{\frac{1}{11}}$$

Converges to  $\frac{1}{3}$ . Telescopic series.

17.) Skip

18.) Alternating Harmonic Series. Converges