CALCULUS BC
WORKSHEET 1 ON 2.1

Work the following on **notebook paper**. Give decimal answers correct to three decimal places.

1. With an initial deposit of \$1000, the balance in a bank account after t years is $f(t) = 1000(1.03)^t$ dollars.

(a) What are the units of the rate of change of $f(t)$?

(b) Compute the average rate of change of $f(t)$ on the following time intervals.

Time interval	$[1.9,\ 2]$	$[1.99,\ 2]$	$[1.999,\ 2]$	$[2,\ 2.1]$	$[2,\ 2.01]$	$[2,\ 2.001]$
Average rate of change						

(c) Use your answers to (b) to estimate the instantaneous rate of change of $f(t)$ at $t = 2$ years.

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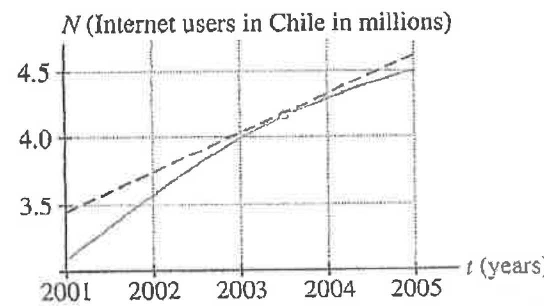
2. The figure on the right shows the estimated number N of internet users in Chile, based on data from the United Nations Statistics Division.

(a) Estimate the rate of change of N at $t = 2003.5$.

(b) Does the rate of change increase or decrease as t increases? Explain.

(c) Compute the average rate of change over [2001, 2005].

(d) Is the rate of change at $t = 2002$ greater than or less than the average rate of change you found in (c)?



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3. Given $f(x) = x^2 - 5x$.

(a) Sketch the function and the tangent line to the graph at the point where $x = 1$.

(b) Based on your sketch, do you expect $f'(1)$ to be positive or negative?

(c) Compute $f'(1)$ three different ways:

1) by using the definition of the derivative
2) by using the alternative form of the definition of the derivative
3) by using the shortcut that you learned last year.

(d) Write the equation of the tangent line to the graph of $y = f(x)$ at $x = 1$. Leave your equation in point-slope form, $y - y_1 = m(x - x_1)$.

TURN->>>

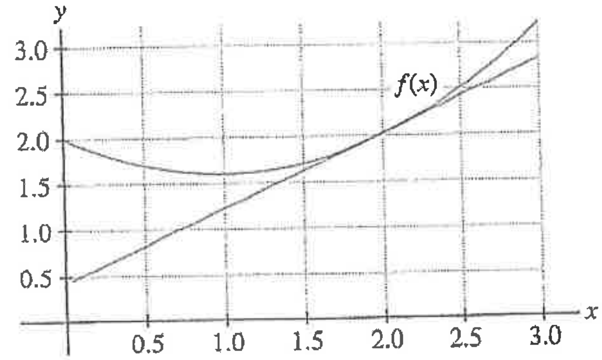
4. Use the figure on the right to answer the following.

(a) Find the slope of the secant line through $(2, f(2))$ and $(2.5, f(2.5))$. Is it larger or smaller than $f'(2)$? Explain.

(b) Estimate $\dfrac{f(2+h)-f(2)}{h}$ for $h=-0.5$. What does this quantity represent? Is it larger or smaller than $f'(2)$

(c) Estimate $f'(1)$ and $f'(2)$.

(d) Find a value of h for which $\dfrac{f(2+h)-f(2)}{h}=0$.



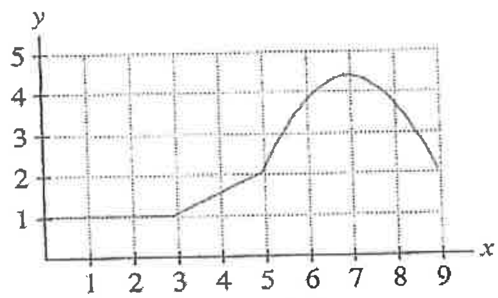
5. Use the figure on the right to answer the following.

(a) Determine $f'(a)$ for $a=1, 2, 4, 7$.

(b) For which values of x is $f'(x)<0$?

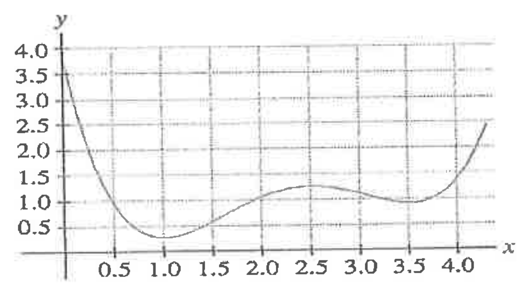
(c) Which is larger, $f'(5.5)$ or $f'(6.5)$?

(d) Do you think that $f'(3)$ exists? Explain.



6. Sketch the graph of $f(x)=\sin x$ on $[0, 2\pi]$, and guess the value of $f'\left(\dfrac{\pi}{2}\right)$. Then calculate the difference quotient at $x=\dfrac{\pi}{2}$ for two small positive and negative values of h. Are these calculations consistent with your guess?

7. Use the graph of the function on the right to determine the intervals along the x-axis on which the derivative is positive.



Use the definition of the derivative, $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$, to find $f'(x)$ on problems 8 – 10.

8. $f(x)=2x^3+5x^2$

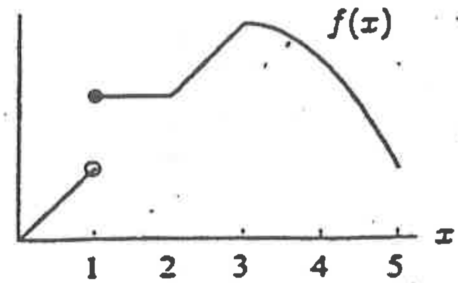
9. $f(x)=\dfrac{1}{x+3}$

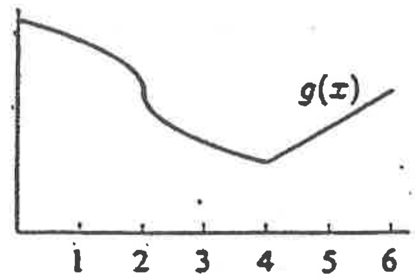
10. $f(x)=\sqrt{x+4}$

CALCULUS BC
WORKSHEET 2 ON 2.1

Work problems 6, 7, and 8 on notebook paper.

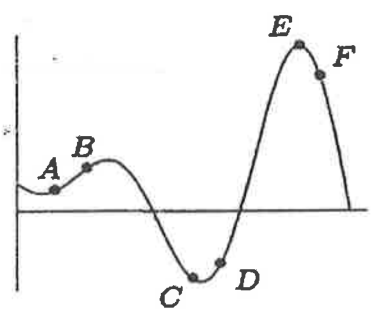
1. For each of the graphs shown below, list the x-values for which the function appears to be:
   (i) not continuous (ii) not differentiable



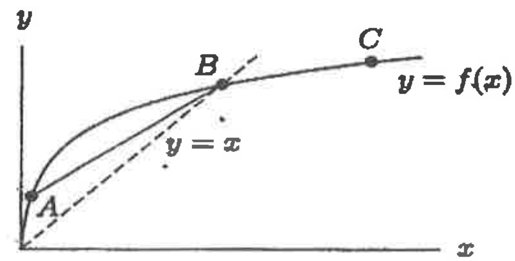


2. Match the points labeled on the curve with the given slopes.

Slope	Point
-3	
-1	
0	
$\frac{1}{2}$	
1	
2	



3. For the graph $y = f(x)$ shown in the figure, arrange the following numbers in ascending order:
   - The slope of the graph at A
   - The slope of the graph at B
   - The slope of the graph at C
   - The slope of the line AB
   - The number 0
   - The number 1

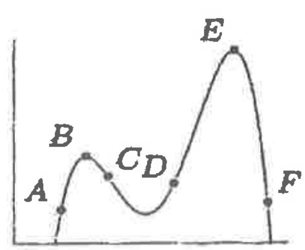


4. For the function shown below, at what labeled points is the slope of the graph:
   positive?

   negative?

   At which labeled point does the graph have the:
   greatest (i.e., most positive) slope?

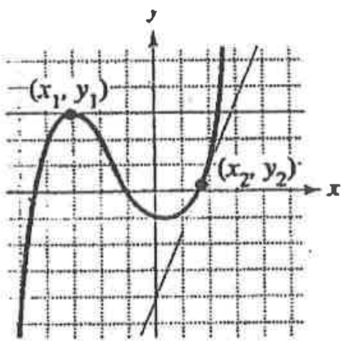
   the least slope (i.e., negative and with the largest magnitude)?



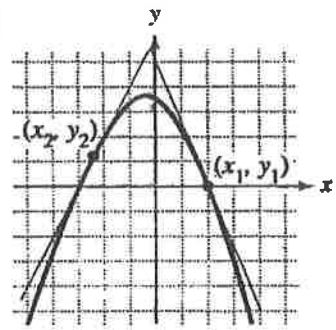
TURN>>>

Estimate the slope of the graph at the points (x_1, y_1) and (x_2, y_2).

5. (a)



(b)



Work problems 6, 7, and 8 on <u>notebook paper</u>.

6. Let f be a function which satisfies the property $f(x+y) = f(x) + f(y) + 2xy$ for all real numbers x and y, and suppose that $\lim_{h \to 0} \frac{f(h)}{h} = 7$. Use the definition of the derivative to find $f'(x)$.

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7. Let f be a function which satisfies $f(1+h) - f(1) = 3h + 4h^2 - 5h^3$ for all real numbers h. Find $f'(1)$.

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8. Suppose f is a function for which $\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = 0$. Which of the following questions <u>**MUST**</u> be true, <u>**MIGHT**</u> be true, or can <u>**NEVER**</u> be true? Explain your answers.

(a) $f(x)$ is continuous at $x = 2$.

(b) $f(x)$ is continuous at $x = 0$.

(c) $f(2) = 0$

(d) $\lim_{x \to 2} f(x) = f(2)$.

(e) $f'(2) = 2$

CALCULUS BC
WORKSHEET ON DERIVATIVES USING DATA

Work the following on **notebook paper.** Give decimal answers correct to **three** decimal places.

1. A roast turkey is taken from an oven and placed on a counter to cool. The table shows the temperature of the turkey at various times over a three hour period.

t (minutes)	0	30	60	90	120	150	180
$T(t)$ (° F)	185	150	130	110	95	88	79

(a) Use the data in the table to find $T(90)$. Using appropriate units, explain the meaning of your answer.

(b) Use the data in the table to find $T^{-1}(150)$. Using appropriate units, explain the meaning of your answer.

(c) For $0 < t < 180$, must there be a time t when the temperature of the turkey is 125° F? Justify your answer.

(d) Use the data in the table to find an approximation for $T'(75)$. Show the computations that lead to your answer. Using appropriate units, explain the meaning of your answer.

2. The number of locations of a popular coffeehouse chain is modeled by a differentiable function N for $0 \le t \le 8$, where t is the number of years since 1996 and $N(t)$ is the number of locations. Values of $N(t)$ at selected values of t are shown in the table below.

t (years)	0	2	4	6	8
$N(t)$ locations	1015	1886	3300	4617	5824

(a) Use the data in the table to find $N(2)$ Using appropriate units, explain the meaning of your answer.

(b) Use the data in the table to find $N^{-1}(4617)$. Using appropriate units, explain the meaning of your answer.

(c) Use the data in the table to find an approximation for $N'(5)$. Show the computations that lead to your answer. Using appropriate units, explain the meaning of your answer.

3. A hot cup of coffee is taken into a classroom and set on a desk to cool. The temperature of the coffee is modeled by a differentiable function T for $0 \le t \le 12$, where t is measure in minutes and $T(t)$ is measured in degrees Fahrenheit. Values of $T(t)$ at selected values of t are shown in the table below.

t (minutes)	0	2	5	6	8	10	12
$T(t)$ (° F)	113	103	95	94	92	91	90

(a) For $0 < t < 12$, must there be a time t when the temperature of the coffee is 99° F? Justify your answer.

(b) Use the data in the table to find an approximation for $T'(3)$. Show the computations that lead to your answer. Using appropriate units, explain the meaning of your answer.

(c) Use the data in the table to find the average rate of change of $T(t)$ on the time period $5 \le t \le 12$ minutes. Show the computations that lead to your answer.

(d) A model for the temperature is given by $y(t) = \frac{1}{3}\left(270 + 70e^{-0.3t}\right)$, where $y(t)$ is measured in degrees Fahrenheit and t is measured in minutes. Find $y'(3)$.

(e) Use the model given in part (d) to find the average rate of change of $y(t)$ on the time period $5 \le t \le 12$ minutes.

TURN->>>

4. The temperature of the water in a pond is modeled by a differentiable function T for $0 \le t \le 15$, where t is measure in days and $W(t)$ is measured in degrees Celsius. Values of $W(t)$ at selected values of t are shown in the table below.

t (days)	0	3	6	9	12	15
$W(t)$ (°C)	20	31	28	24	22	20

(a) Use data from the table to find $W(9)$. Using appropriate units, explain the meaning of your answer.

(b) Use data from the table to find $W^{-1}(31)$. Using appropriate units, explain the meaning of your answer.

(c) Use data from the table to find an approximation for $W'(7)$. Show the computations that lead to your answer.

(d) Based on values in the table, what is the smallest number of instances at which the temperature of the pond could equal 23°C on the open interval $0 < t < 15$? Justify your answer.

(e) A student proposes the function P, given by $P(t) = 20 + 10te^{\left(-\frac{t}{3}\right)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(7)$. Using appropriate units, explain the meaning of your answer.

5. A car travels on a straight road. During the time interval $0 \le t \le 60$ the car's velocity $v(t)$, measured in feet per second, is a twice-differentiable function. The table below shows selected values of this function.

t (sec)	0	14	24	33	47	60
$v(t)$ (ft/sec)	18	25	30	21	15	17

(a) Use data from the table to find an approximation for $v'(17)$. Show the computations that lead to your answer. Using appropriate units, explain the meaning of your answer.

(b) Based on the values in the table, what is the smallest number of instances at which the velocity of the car could be 20 feet per second on the open interval $0 < t < 60$? Justify your answer.

6. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table below.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.2

(a) Use data from the table to find an approximation for $v'(18)$. Show the computations that lead to your answer. Using appropriate units, explain the meaning of your answer.

(b) Use data from the table to find the average acceleration of the particle over the time interval $5 \le t \le 25$ minutes. (Hint: The average acceleration is the average rate of change of the velocity.)

(c) Based on the values in the table, what is the smallest number of instances at which the velocity of the plane could be 8 miles per minute on the open interval $0 < t < 40$? Justify your answer.

(d) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.