Particle Motion

Typically, if a particle is moving along the x-axis at any time, t, $x(t)$ represents the position of the particle; along the y-axis, $y(t)$ is often used; along another straight line, $s(t)$ is often used. In addition, $v(t)$ is typically used to represent the velocity of the particle. In these types of particle motion problems,

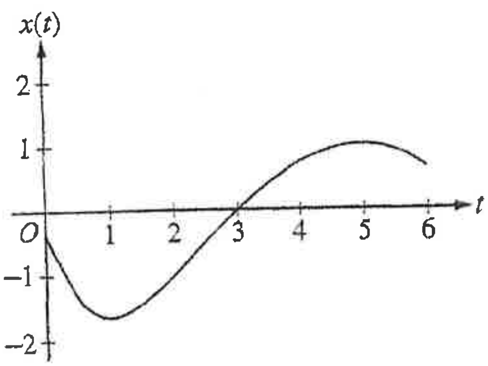
- "Initially" means when time $t = 0$.
- "At the origin" means $x(t) = 0$.
- "At rest" means velocity $v(t) = 0$.
- Average velocity of the particle is $\dfrac{\Delta s}{\Delta t} = \dfrac{s(t_2) - s(t_1)}{t_2 - t_1}$.
- If the velocity of the particle is positive, then the particle is moving to the right.
- If the velocity of the particle is negative, then the particle is moving to the left.
- If the acceleration of the particle is positive, then the velocity is increasing.
- If the acceleration of the particle is negative, then the velocity is decreasing.
- Speed is the absolute value of velocity.
- If the velocity and acceleration have the same sign (both positive or both negative), then speed is increasing.
- If the velocity and acceleration are opposite in sign (one is positive and the other is negative), then speed is decreasing.
- To determine total distance traveled over a time interval, you must calculate the sum of the absolute values of the differences in position between all resting points or calculate the area under the absolute value of the velocity curve, $\int_{t_1}^{t_2} |v(t)|\, dt$.
- Displacement can be determined using $displacement = \int_{t_1}^{t_2} v(t)\, dt$
- To determine the final position of a particle after motion use $s(t_2) = s(t_1) + \int_{t_1}^{t_2} v(t)\, dt$

Multiple Choice

1. (calculator not allowed)
A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only
(B) $t = 3$ only
(C) $t = \frac{7}{2}$ only
(D) $t = 3$ and $t = \frac{7}{2}$
(E) $t = 3$ and $t = 4$

2. (calculator not allowed)



A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

3. (calculator not allowed)

A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at time $t - 1$?

(A) 4
(B) 6
(C) 0
(D) 11
(E) 12

4. (calculator not allowed)

The velocity of a particle moving on a line at time t is $v = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ meters per second. How many meters did the particle travel from $t = 0$ to $t = 4$?

(A) 32
(B) 40
(C) 64
(D) 80
(E) 184

5. (calculator not allowed)

If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0 \leq t \leq 3$ is

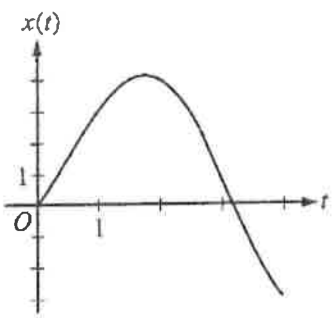
(A) –45
(B) –30
(C) –15
(D) –10
(E) –5

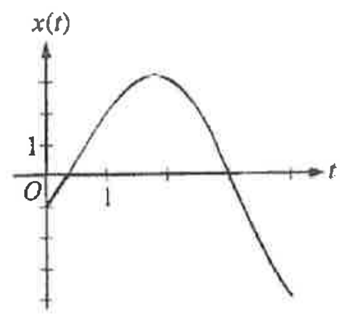
6. (calculator not allowed)
A particle travels in a straight line with a constant acceleration of 3 meters per second per second. If the velocity of the particle is 10 meters per second at time 2 seconds, how far does the particle travel during the time interval when its velocity increases from 4 meters per second to 10 meters per second?

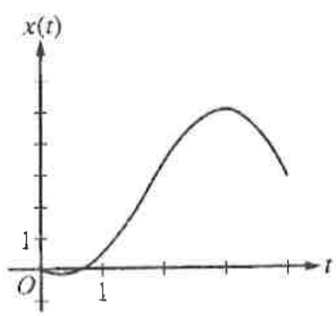
(A) 20 m
(B) 14 m
(C) 7 m
(D) 6 m
(E) 3 m

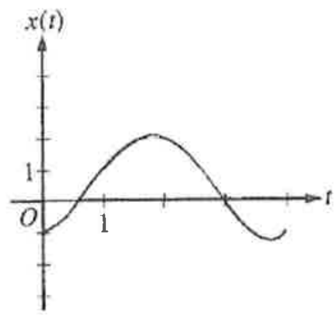
7. (calculator not allowed)
The table gives selected values of the velocity, $v(t)$, of a particle moving along the x-axis. At time $t = 0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \le t \le 4$?

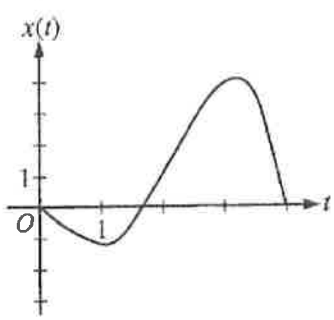
t	0	1	2	3	4
$v(t)$	−1	2	3	0	−4

(A)


(B)


(C)


(D)


(E)


8. (calculator allowed)
A particle moves along the x- axis so that any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

(A) -2.016
(B) -0.677
(C) 1.633
(D) 1.814
(E) 2.978

9. (calculator allowed)
A particle moves along the x-axis so that any time $t > 0$, its acceleration is given by $a(t) = \ln(1 + 2^t)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

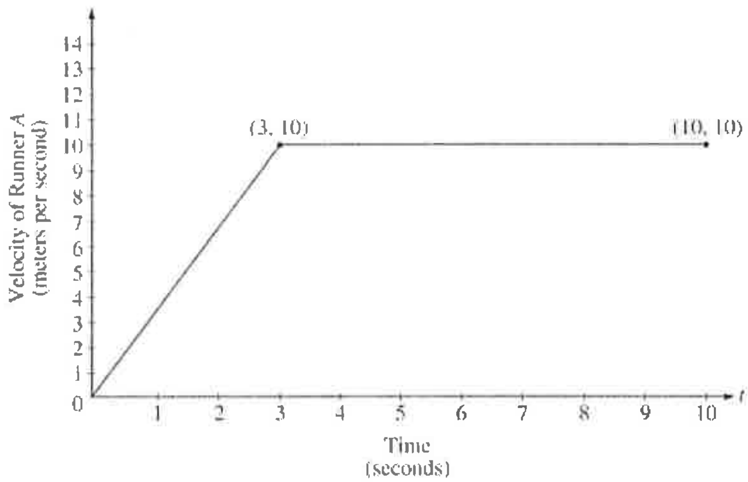
(A) 0.462
(B) 1.609
(C) 2.555
(D) 2.886
(E) 3.346

10. (calculator allowed)
The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

(A) 20.086 ft/sec
(B) 26.447 ft/sec
(C) 32.809 ft/sec
(D) 40.671 ft/sec
(E) 79.342 ft/sec

Free Response

11. (calculator allowed)



Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.

(a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

(b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

(c) Find the total distance run by Runner A and the total distance run by Runner B over the interval $0 \le t \le 10$ seconds. Indicate units of measure.

12. (calculator allowed)

A particle moves along the y-axis with velocity given by $v(t) = t\sin\left(t^2\right)$ for $t \geq 0$.

(a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

(b) Find the acceleration of the particle at time $t = 1.5$? Is the velocity of the particle increasing at time $t = 1.5$?

(c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

13. (calculator allowed)

A particle moves along the x-axis so that its velocity v at time t, $0 \le t \le 5$, is given by $v(t) = \ln\left(t^2 - 3t + 3\right)$. The particle is at position $x = 8$ at time $t = 0$.

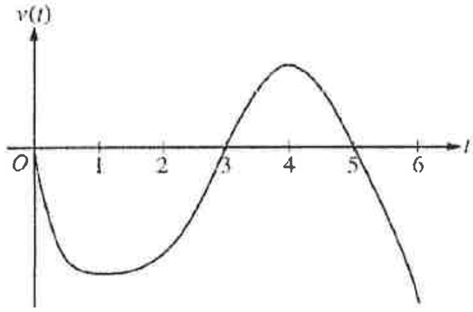
(a) Find the acceleration of the particle at time $t = 4$.

(b) Find the times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, $0 < t < 5$, does the particle travel left?

(c) Find the position of the particle at time $t = 2$.

(d) Find the average speed of the particle over the interval $0 \le t \le 2$.

14. (calculator not allowed)



Graph of v

A particle move along the x-axis so that the velocity at time, t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals [0, 3], [3, 5], and [5, 6] are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

15.

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

t *(seconds)*	0	10	40	60
$B(t)$ *(meters)*	100	136	9	49
$v(t)$ $\left(\frac{m}{\text{sec}}\right)$	2.0	2.3	2.5	4.6

(a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\,dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\,dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

(c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.