CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. If $f(x) = \dfrac{x^2 + 3x + 2}{x + 3}$, then $f'(x) =$

(A) $2x + 3$

(B) $\dfrac{-x^2 - 6x - 7}{(x + 3)^2}$

(C) $\dfrac{x^2 + 6x + 7}{(x + 3)^2}$

(D) $\dfrac{x^2 + 12x + 11}{(x + 3)^2}$

(E) $\dfrac{3x^2 + 12x + 11}{(x + 3)^2}$

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2. $\int 5x\left(\sqrt{x} - x^2\right) dx =$

(A) $\dfrac{15\sqrt{x}}{2} - 15x^2 + C$

(B) $5x - \dfrac{5x^4}{4} + C$

(C) $2x^{5/2} - \dfrac{5x^4}{4} + C$

(D) $\dfrac{25x^{5/2}}{2} - \dfrac{5x^4}{4} + C$

(E) $\dfrac{5x^{7/2}}{3} - \dfrac{5x^6}{6} + C$

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3. What is the value of $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{5^n}$?

(A) $-\frac{15}{8}$ (B) $-\frac{9}{8}$ (C) $-\frac{3}{8}$ (D) $\frac{9}{8}$ (E) $\frac{15}{8}$

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4. Which of the following is an equation of the line tangent to the graph of $x^2 - 3xy = 10$ at the point $(1, -3)$?

(A) $y + 3 = -11(x - 1)$

(B) $y + 3 = -\frac{7}{3}(x - 1)$

(C) $y + 3 = \frac{1}{3}(x - 1)$

(D) $y + 3 = \frac{7}{3}(x - 1)$

(E) $y + 3 = \frac{11}{3}(x - 1)$

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5. If $y = \frac{1}{2}x^{4/5} - \frac{3}{x^5}$, then $\frac{dy}{dx} =$

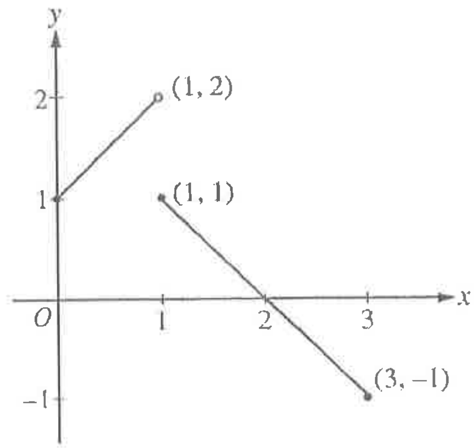
(A) $\frac{2}{5x^{1/5}} + \frac{15}{x^6}$

(B) $\frac{2}{5x^{1/5}} + \frac{15}{x^4}$

(C) $\frac{2}{5x^{1/5}} - \frac{3}{5x^4}$

(D) $\frac{2x^{1/5}}{5} + \frac{15}{x^6}$

(E) $\frac{2x^{1/5}}{5} - \frac{3}{5x^4}$



Graph of f

6. The graph of the function f consists of two line segments, as shown in the figure above. The value of $\int_0^3 |f(x)|\,dx$ is

(A) $-\frac{3}{2}$ (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $\frac{5}{2}$ (E) nonexistent

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7. A population y changes at a rate modeled by the differential equation $\frac{dy}{dt} = 0.2y(1000 - y)$, where t is measured in years. What are all values of y for which the population is increasing at a decreasing rate?

(A) 500 only

(B) $0 < y < 500$ only

(C) $500 < y < 1000$ only

(D) $0 < y < 1000$

(E) $y > 1000$

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8. Which of the following gives the length of the path described by the parametric equations $x(t) = 2 + 3t$ and $y(t) = 1 + t^2$ from $t = 0$ to $t = 1$?

(A) $\int_0^1 \sqrt{1 + \frac{4t^2}{9}}\, dt$

(B) $\int_0^1 \sqrt{1 + 4t^2}\, dt$

(C) $\int_0^1 \sqrt{3 + 3t + t^2}\, dt$

(D) $\int_0^1 \sqrt{9 + 4t^2}\, dt$

(E) $\int_0^1 \sqrt{(2 + 3t)^2 + \left(1 + t^2\right)^2}\, dt$

9. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2x + y$ with initial condition $f(1) = 0$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length, starting at $x = 1$?

(A) 0 (B) 1 (C) 2.75 (D) 3 (E) 6

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10. If $\int_0^k \frac{x}{x^2 + 4}\,dx = \frac{1}{2}\ln 4$, where $k > 0$, then $k =$

(A) 0 (B) $\sqrt{2}$ (C) 2 (D) $\sqrt{12}$ (E) $\frac{1}{2}\tan\left(\ln\sqrt{2}\right)$

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11. The third-degree Taylor polynomial for a function f about $x = 4$ is $\dfrac{(x-4)^3}{512} - \dfrac{(x-4)^2}{64} + \dfrac{(x-4)}{4} + 2$. What is the value of $f'''(4)$?

(A) $-\dfrac{1}{64}$ (B) $-\dfrac{1}{32}$ (C) $\dfrac{1}{512}$ (D) $\dfrac{3}{256}$ (E) $\dfrac{81}{256}$

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12. For which of the following does $\lim\limits_{x \to \infty} f(x) = 0$?

I. $f(x) = \dfrac{\ln x}{x^{99}}$

II. $f(x) = \dfrac{e^x}{\ln x}$

III. $f(x) = \dfrac{x^{99}}{e^x}$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

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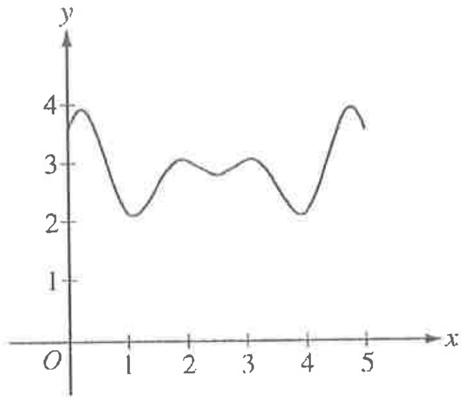

Graph of f'

13. The graph of f', the derivative of f, is shown in the figure above. If $f(0) = 20$, which of the following could be the value of $f(5)$?

(A) 15 (B) 20 (C) 25 (D) 35 (E) 40

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14. If a and b are positive constants, then $\lim_{x\to\infty} \dfrac{\ln(bx+1)}{\ln(ax^2+3)} =$

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{2}ab$ (D) 2 (E) ∞

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15. What are all values of x for which the series $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{n}\left(x+\dfrac{3}{2}\right)^n$ converges?

(A) $-\frac{5}{2} < x < -\frac{1}{2}$

(B) $-\frac{5}{2} < x \le -\frac{1}{2}$

(C) $-\frac{5}{2} \le x < -\frac{1}{2}$

(D) $-\frac{1}{2} < x < \frac{1}{2}$

(E) $x \le -\frac{1}{2}$

16. For $0 < P < 100$, which of the following is an antiderivative of $\dfrac{1}{100P - P^2}$?

(A) $\dfrac{1}{100}\ln(P) - \dfrac{1}{100}\ln(100 - P)$

(B) $\dfrac{1}{100}\ln(P) + \dfrac{1}{100}\ln(100 - P)$

(C) $100\ln(P) - 100\ln(100 - P)$

(D) $\ln\left(100P - P^2\right)$

(E) $\dfrac{1}{50P^2 - \dfrac{P^3}{3}}$

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17. If $\lim\limits_{h \to 0} \dfrac{\arcsin(a + h) - \arcsin(a)}{h} = 2$, which of the following could be the value of a ?

(A) $\dfrac{\sqrt{2}}{2}$ (B) $\dfrac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\dfrac{1}{2}$ (E) 2

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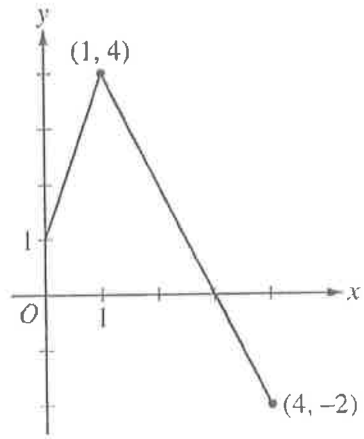

Graph of f

18. The graph of the function f, consisting of two line segments, is shown in the figure above. Let g be the function given by $g(x) = 2x + 1$, and let h be the function given by $h(x) = f(g(x))$. What is the value of $h'(1)$?

(A) -4 (B) -2 (C) 4 (D) 6 (E) nonexistent

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19. Which of the following is the Maclaurin series for $\dfrac{1}{(1-x)^2}$?

(A) $1 - x + x^2 - x^3 + \cdots$

(B) $1 - 2x + 3x^2 - 4x^3 + \cdots$

(C) $1 + 2x + 3x^2 + 4x^3 + \cdots$

(D) $1 + x^2 + x^4 + x^6 + \cdots$

(E) $x + \dfrac{x^2}{2} + \dfrac{x^3}{3} + \dfrac{x^4}{4} + \cdots$

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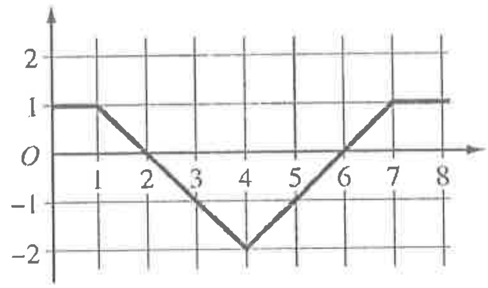

Graph of f

20. The graph of the function f in the figure above consists of four line segments. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. Which of the following is an equation of the line tangent to the graph of g at $x = 5$?

(A) $y + 1 = x - 5$

(B) $y - 2 = x - 5$

(C) $y - 2 = -1(x - 5)$

(D) $y + 2 = x - 5$

(E) $y + 2 = -1(x - 5)$

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21. At time $t \geq 0$, a cube has volume $V(t)$ and edges of length $x(t)$. If the volume of the cube decreases at a rate proportional to its surface area, which of the following differential equations could describe the rate at which the volume of the cube decreases?

(A) $\dfrac{dV}{dt} = -1.2x^2$

(B) $\dfrac{dV}{dt} = -1.2x^3$

(C) $\dfrac{dV}{dt} = -1.2x^2t$

(D) $\dfrac{dV}{dt} = -1.2t^2$

(E) $\dfrac{dV}{dt} = -1.2V^2$

22. Which of the following is true about the curve $x^2 - xy + y^2 = 3$ at the point $(2, 1)$?

(A) $\frac{dy}{dx}$ exists at $(2, 1)$, but there is no tangent line at that point.

(B) $\frac{dy}{dx}$ exists at $(2, 1)$, and the tangent line at that point is horizontal.

(C) $\frac{dy}{dx}$ exists at $(2, 1)$, and the tangent line at that point is neither horizontal nor vertical.

(D) $\frac{dy}{dx}$ does not exist at $(2, 1)$, and the tangent line at that point is vertical.

(E) $\frac{dy}{dx}$ does not exist at $(2, 1)$, and the tangent line at that point is horizontal.

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23. What is the coefficient of x^6 in the Taylor series for $\frac{e^{3x^2}}{2}$ about $x = 0$?

(A) $\frac{1}{1440}$ (B) $\frac{81}{160}$ (C) $\frac{9}{4}$ (D) $\frac{9}{2}$ (E) $\frac{27}{2}$

24. The function g is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of g on the closed interval $[-2, 1]$?

(A) -7 (B) $-\frac{3}{4}$ (C) 0 (D) 2 (E) 6

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25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln\left(-e^x + 5\right)$

(D) $y = -\ln\left(e^x + 3\right)$

(E) $y = \ln\left(e^x + 3\right)$

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26. Which of the following series converge?

I. $\displaystyle\sum_{n=1}^{\infty} \frac{|\sin n|}{n^2}$ II. $\displaystyle\sum_{n=1}^{\infty} e^{-n}$ III. $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{n^2+n}$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

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27. If $\displaystyle\int_1^x f(t)\,dt = \frac{20x}{\sqrt{4x^2+21}} - 4$, then $\displaystyle\int_1^{\infty} f(t)\,dt$ is

(A) 6 (B) 1 (C) -3 (D) -4 (E) divergent

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28. If $x = t^2 - 1$ and $y = \ln t$, what is $\dfrac{d^2y}{dx^2}$ in terms of t ?

(A) $-\dfrac{1}{2t^4}$ (B) $\dfrac{1}{2t^4}$ (C) $-\dfrac{1}{t^3}$ (D) $-\dfrac{1}{2t^2}$ (E) $\dfrac{1}{2t^2}$

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.