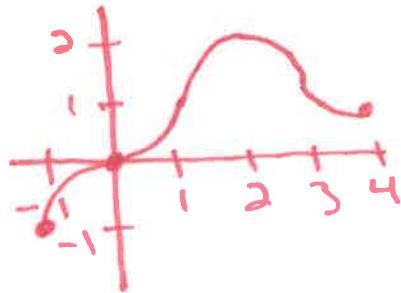


Worksheet on f, f', f''

- 1.)
- a.) $x=2$. $f'(x)$ changes from + to - at $x=2$.
- b.) Since f is continuous, it must have an absolute min. There are no relative mins since f' never crosses the x -axis, so the abs min must occur at $x=-1$ or $x=4$.
 $f(-1) = -1$ and $f(4) = 1$ so the abs min occurs at $x=-1$.
- c.) f is CD when f' is decreasing.
 $(-1, 0) \cup (1, 3)$.
- d.) f has a POI at $x=0$ b/c f' changes from dec. to inc. also at $x=1$ b/c f' changes from inc. to dec. and at $x=3$ b/c f' changes from dec. to inc.

e.)



2.) a.) Abs. max at $x=-1$ Abs min at $x=3$

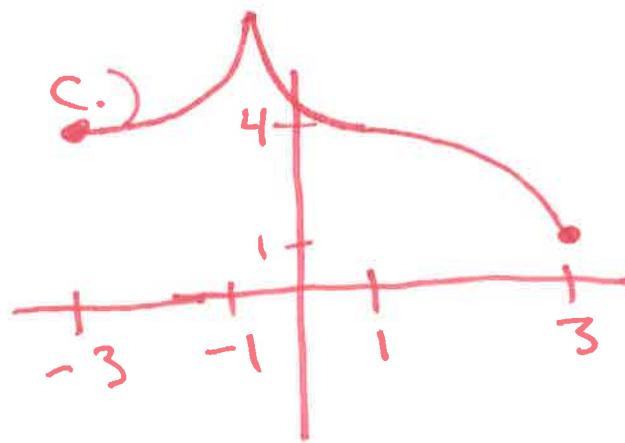
x	$f(x)$
-3	4
-1	Higher than 4
3	1

C.P. \rightarrow -1

The abs. max occurs at $x=-1$. This is the only C.P. and it increases from $x=-3$ to $x=-1$ then dec. to $x=3$. There are no rel. min., so the abs. min occurs at $x=-3$ or $x=3$.

$$f(3)=1, f(-3)=4.$$

b.) $x=1$ is a P.O.I. b/c f'' changes sign at $x=1$ from + to -.



$$\begin{array}{ccccccc} 3.) f & \text{CU} & \text{CD} & \text{CU} & \text{CU} \\ & \leftarrow & \rightarrow & \leftarrow & \rightarrow \\ f'' & + & -1 & -0 & + & 2 & + \\ \boxed{C} & & & & & & \end{array}$$

f has inflection points at $x=-1$ and $x=0$ b/c f'' changes sign.

$$\begin{aligned} 4.) \quad y' &= 3x^2 + 6x & y'' &= 6x + 6 = 0 \text{ when } x = -1 \\ y'(-1) &= -3 & (-1, 4) & \leftarrow \text{CD} \rightarrow \\ y-4 &= -3(x+1) \rightarrow y = 4-3x-3 \rightarrow y = -3x+1 & y'' &= -6 & + \end{aligned}$$

\boxed{B}

5.) $y' = 3x^2 + 2ax + b$ $y'' = 6x + 2a$
 $y'' = 0 \text{ when } x=1 \rightarrow 6x + 2a = 0 \rightarrow 6 + 2a = 0 \rightarrow a = -3$
 $-6 = 1^3 + (-3)(1)^2 + b - 4 \rightarrow -6 = 1 - 3 + b - 4 \rightarrow b = 0$

B

6.) $f(x) = (x-2)(x-3)^2$

$$f'(x) = 2(x-3)(x-2) + (x-3)^2$$

$$f'(x) = (2x-4)(x-3) + (x-3)^2 = (x-3)(2x-4+x-3)$$

$$= (x-3)(3x-7)$$

$$\begin{array}{c} f \leftarrow \\ f' \leftarrow \end{array} \begin{array}{c} \nearrow \\ + \end{array} \begin{array}{c} \nearrow \\ 7/3 \end{array} \begin{array}{c} \nearrow \\ - \end{array} \begin{array}{c} \nearrow \\ 3x \end{array}$$

f has a rel. max at $7/3$ b/c f' changes from $+$ to $-$.

D

7.) E

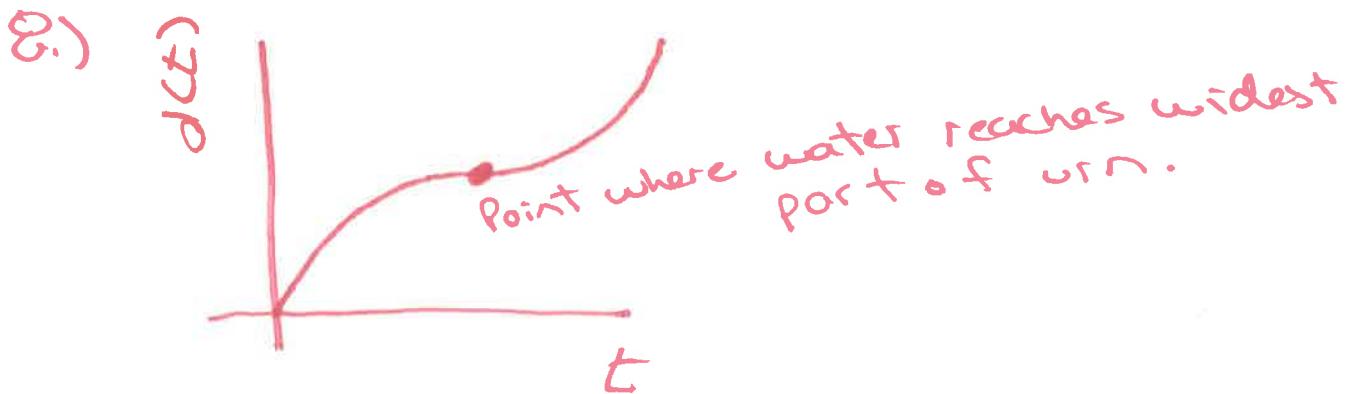
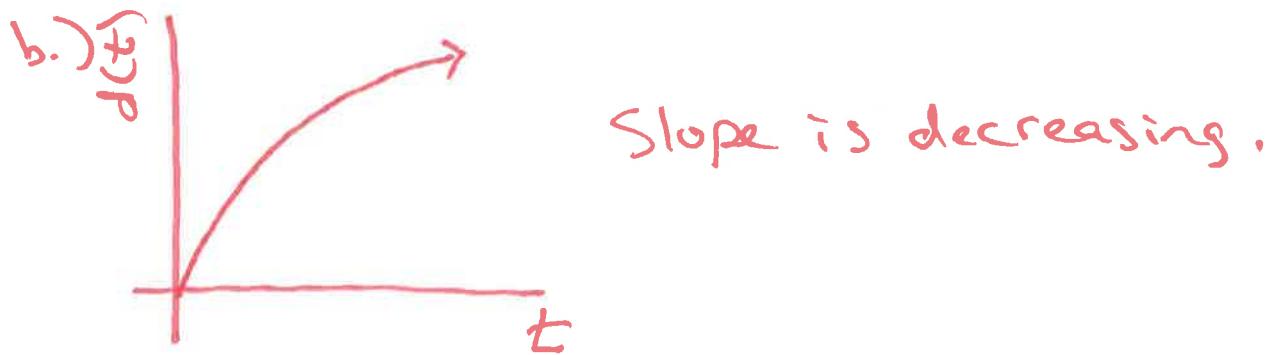
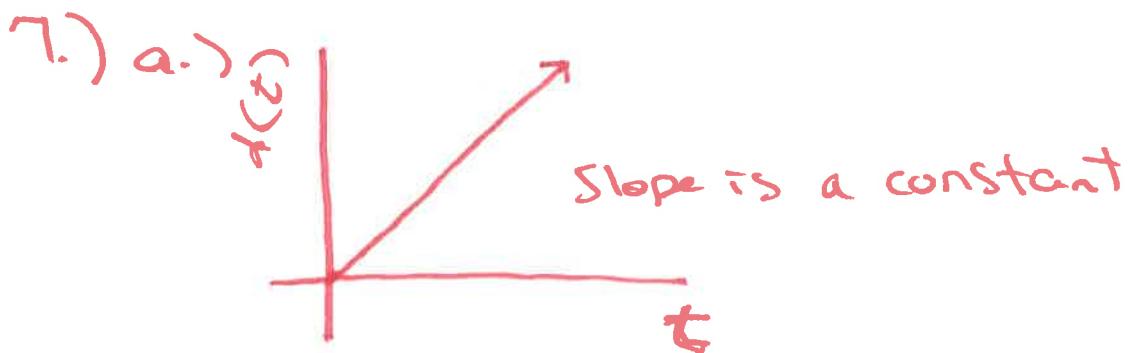
8.) $\frac{f(b) - f(a)}{b-a} = f'(c)$

$$\frac{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}}{\pi} = \frac{1}{2} \cos\left(\frac{\pi}{2}\right)$$

$$0 = \frac{1}{2} \cos\left(\frac{\pi}{2}\right) \quad x = \pi$$

D

- 1.) Inc. & CU $f' > 0$ $f'' > 0$ w.s. #2
- 2.) Constant $f' = 0$ $f'' = 0$
- 3.) Dec. & linear $f' < 0$ $f'' = 0$
- 4.) Dec. & CU $f' < 0$ $f'' > 0$
- 5.) Inc. & CD $f' > 0$ $f'' < 0$
- 6.) Dec. & CD $f' < 0$ $f'' < 0$



9.) Polynomials are diff. everywhere
since $f(a) = f(b) = 1$, $f'(x) = 0$ somewhere
between a & b . Rolle's theorem.

C

10.) $f'(x) = 3x^2 - 6x \rightarrow 3x(x-2) = 0$
C.P. at $x=0$ & $x=2$

x	$f(x)$
-2	-8
0	12
2	8
4	28

A

11.) $y = x^{-2} - x^{-3}$ $y' = -2x^{-3} + 3x^{-4}$ $y'' = 6x^{-4} - 12x^{-5}$
 $y'' = \frac{6}{x^4} - \frac{12}{x^5} = 0$ $\frac{6}{x^4} = \frac{12}{x^5} \rightarrow 6x^5 = 12x^4$

$6x^5 - 12x^4 = 0 \rightarrow 6x^4(x-2) = 0$ when $x=0, 2$

$$\begin{array}{ccccccc} y & \leftarrow & \text{CD} & \text{CD} & \rightarrow & \text{CD} & \\ y'' = 0 & - & 2 & + & & & \end{array}$$

y has a P.O.I. at
 $x=2$ b/c y'' changes
sign

C

12.) B

13.) A

14.) Increasing with a decreasing slope.

B

15.) A