1.) $y\,dy = (x-3)\,dx \qquad (2,-5)$ I.C.

$$\int y\,dy = \int (x-3)\,dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 - 3x + C$$

$$\frac{1}{2}(-5)^2 = \frac{1}{2}(2)^2 - 3(2) + C$$

$$\frac{25}{2} = -4 + C \longrightarrow C = \frac{33}{2}$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 - 3x + \frac{33}{2}$$

$$y^2 = x^2 - 6x + 33$$

$$y = \pm\sqrt{x^2 - 6x + 33}$$

since $y(2) = -5$

$$y = -\sqrt{x^2 - 6x + 33}$$

2) $\frac{dy}{dx} = 2x\sqrt{y}$ $(2, 25)$ I.C

$\frac{dy}{\sqrt{y}} = 2x\,dx$

$\int y^{-1/2}\,dy = \int 2x\,dx$

$2\sqrt{y} = x^2 + c$

$2\sqrt{25} = 2^2 + c \rightarrow 10 = 4 + c \rightarrow c = 6$

$2\sqrt{y} = x^2 + 6$

$\sqrt{y} = \frac{1}{2}x^2 + \frac{6}{2} \rightarrow y = \left(\frac{1}{2}x^2 + 3\right)^2$

3) $\dfrac{dy}{y^2} = 4\sec^2(2x)dx \quad (\frac{\pi}{8}, 1)$

I. C.

$\int y^{-2}dy = \int 4\sec^2(2x)dx$

Note

$-\dfrac{1}{y} = 2\tan(2x) + C \qquad \int \sec^2(u)du = \tan(u)+C$

$\dfrac{-1}{1} = 2\tan(2\cdot\frac{\pi}{8}) + C$

$-1 = 2\tan(\frac{\pi}{4}) + C \rightarrow -1 = 2(1) + C$

$C = -3$

$\dfrac{-1}{y} = 2\tan(2x) - 3$

$\dfrac{-1}{2\tan(2x)-3} = y$

4.) $y\,dy = \frac{1}{x}\ln x\,dx$ $\quad (1,2)$ I.C.

$$\int y\,dy = \int \frac{1}{x}\ln x\,dx$$

Note

$\int u\,du = \frac{1}{2}u^2 + c$

$u = \ln x$

$du = \frac{1}{x}dx$

Perfect Integral!

$$\frac{1}{2}y^2 = \frac{1}{2}(\ln x)^2 + c$$

$$\frac{1}{2}(2)^2 = \frac{1}{2}(\ln 1)^2 + c$$

$$2 = \frac{1}{2}(0) + c \rightarrow c = 2$$

$$\frac{1}{2}y^2 = \frac{1}{2}(\ln x)^2 + 2$$

$$y^2 = (\ln x)^2 + 4 \rightarrow y = \pm\sqrt{(\ln x)^2 + 4}$$

Since $y(1) = 2$

$$y = \sqrt{(\ln x)^2 + 4}$$

5.) $\frac{dy}{dx} = 2x \sec y$ $(2, -\pi/2)$

I.C.

$\frac{dy}{\sec y} = 2x\,dx$

Note

$\frac{1}{\sec y} = \cos y$

$\int \cos y\,dy = \int 2x\,dx$

$\sin y = x^2 + C$

$\sin(-\pi/2) = 2^2 + C \rightarrow -1 = 4 + C \rightarrow C = -5$

$\sin y = x^2 - 5$

$y = \sin^{-1}(x^2 - 5)$

6.) $\frac{dy}{dx} = xe^{y} + 2e^{y}$

(0, 0)
I. C

$\frac{dy}{dx} = e^{y}(x+2)$

Note
we must factor before we separate variables

$\frac{dy}{e^{y}} = (x+2)dx$

$\int e^{-y} dy = \int (x+2) dx$

Note
$\int e^{u} du = e^{u} + c$

$-e^{-y} = \frac{1}{2}x^{2} + 2x + c$

$u = -y$
$-du = dy$

$-e^{0} = \frac{1}{2}(0)^{2} + 2(0) + c$

$-1 = c$

$-e^{-y} = \frac{1}{2}x^{2} + 2x - 1$

$\frac{1}{e^{y}} = -\frac{1}{2}x^{2} - 2x + 1$

$\frac{1}{-\frac{1}{2}x^{2} - 2x + 1} = e^{y} \rightarrow y = \ln\left(\frac{1}{-\frac{1}{2}x^{2} - 2x + 1}\right)$

7.) $\frac{dy}{y^3} = 2x\sin(x^2)\,dx$ $(0,-1)$

I.C.

$\int y^{-3}\,dy = \int 2x\sin(x^2)\,dx$

Note

$\int \sin(u)\,du = -\cos u + C$

$u = x^2$

$du = 2x\,dx$

Perfect!

$\frac{-1}{2y^2} = -\cos(x^2) + C$

$\frac{-1}{2(-1)^2} = -\cos(0) + C$

$\frac{-1}{2} = -1 + C \rightarrow C = \frac{1}{2}$

$\frac{-1}{2y^2} = -\cos(x^2) + \frac{1}{2}$

$\frac{1}{y^2} = 2\cos(x^2) - 1$

$y^2 = \frac{1}{2\cos(x^2)-1}$

$y = \pm\sqrt{\frac{1}{2\cos(x^2)-1}}$

since $y(0) = -1$

$y = -\sqrt{\frac{1}{2\cos(x^2)-1}}$

8.) $y^2 dy = 1 dx$ $(0,4)$ I.C.

$\int y^2 dy = \int dx$

$\frac{y^3}{3} = x + C$

$\frac{4^3}{3} = 0 + C \rightarrow \frac{64}{3} = C$

$\frac{y^3}{3} = x + \frac{64}{3}$

$y^3 = 3x + 64$

$y = \sqrt[3]{3x+64}$

9) $\frac{dy}{dx} = \frac{2x}{y^2}$

$(0,3)$
I.C.

$y^2\,dy = 2x\,dx$

Note
If the tangent line has a slope of "blah" at (x,y) then so does the curve!

$\int y^2\,dy = \int 2x\,dx$

$\frac{1}{3}y^3 = x^2 + C$

$\frac{1}{3}(3)^3 = (0)^2 + C$

$9 = C$

$\frac{1}{3}y^3 = x^2 + 9$

$y^3 = 3x^2 + 27$

$y = \sqrt[3]{3x^2 + 27}$