

# 7.1 Solutions

1)  $x^2 + 2x + 1 = 3x + 3 \rightarrow x^2 - x - 2 = 0 \quad x = -1 \text{ and } x = 2$



~~$$A = \int_{-1}^2 (3x+3) - (x^2+2x+1) dx = \frac{9}{2}$$~~

2)  $x^3 - 3x^2 + 3x = x^2 \rightarrow x^3 - 4x^2 + 3x = 0 \quad x = 0 \quad x = 1 \quad x = 3$

~~$$A = \int_0^1 (x^3 - 3x^2 + 3x) - x^2 dx + \int_1^3 x^2 - (x^3 - 3x^2 + 3x) dx = \frac{37}{12}$$~~

3.) 
 $y^2 = y + 2 \rightarrow y^2 - y - 2 = 0 \quad y = -1 \text{ and } y = 2$

~~$$A = \int_{-1}^2 y+2 - y^2 dy = \frac{9}{2}$$~~

4.) 
 $x^4 = 3x + 4 \quad x = -1 \text{ and } x = 1.74296$

~~$$A = \int_{-1}^{1.74296} 3x+4 - x^4 dx = 10.6116$$~~

5)  $x^2 = 2^x \rightarrow x = -0.7666665 \quad x = 2 \quad x = 4$

~~$$A = \int_{-0.7666665}^2 2^x - x^2 dx + \int_2^4 x^2 - 2^x dx = 3.46025$$~~

6)  $\ln(x^2 + 1) = \cos x \quad x = -0.915858 \quad x = 0.915858$

~~$$A = \int_{-0.915858}^{0.915858} \cos x - \ln(x^2 + 1) dx = 1.16785$$~~

$$7.7^{\text{a.)}} A = \int_0^4 \sqrt{x} dx + \int_4^6 6-x dx = \frac{22}{3}$$

$$\text{b) } x = y^2 \quad x = 6-y$$

$$A = \int_0^2 6-y - y^2 dy = \frac{22}{3}$$

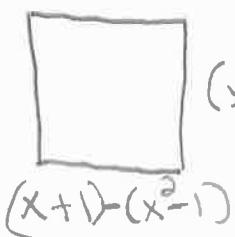
$$\text{c) } y$$

$$8.) 2^x = \frac{1}{x} \quad x = .641186 \quad 4^x = \frac{1}{x} \quad x = .5$$

$$A = \int_0^{.5} 4^x - 2^x dx + \int_{.5}^{.641186} \frac{1}{x} - \frac{1}{x^2} dx = .162711$$

# Volume by Cross Section Solutions

1.) a)



$$(x+1) - (x^2 - 1)$$

$$(x+1) - (x^2 - 1)$$



$$V = \int_{-1}^2 ((x+1) - (x^2 - 1))^2 dx = \frac{81}{10}$$

b)



$$(x+1) - (x^2 - 1)$$

$$V = \int_{-1}^2 (x+1) - (x^2 - 1) dx = 4.5$$

c)



$$(x+1) - (x^2 - 1)$$

$$V = \int_{-1}^2 \frac{1}{2} \pi (2) \left( \frac{x+1 - (x^2 - 1)}{2} \right) dx = \frac{9\pi}{4}$$

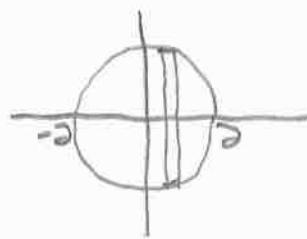
d.)



$$(x+1) - (x^2 - 1)$$

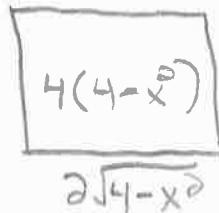
$$V = \frac{\sqrt{3}}{2} \int_{-1}^2 ((x+1) - (x^2 - 1))^2 dx = \frac{81\sqrt{3}}{40}$$

2.)



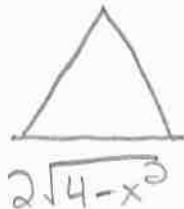
$$y = \pm \sqrt{4 - x^2}$$

a.)



$$V = \int_{-2}^2 16 - 4x^2 dx = \frac{128}{3}$$

b.)



$$V = \frac{\sqrt{3}}{4} \int_{-2}^2 16 - 4x^2 dx = \frac{128\sqrt{3}}{15}$$

c.)



$$V = \frac{\pi}{2} \int_{-2}^2 4 - x^2 dx = \frac{16\pi}{3}$$

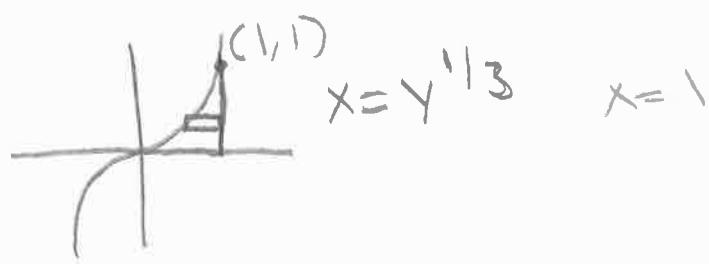
d.)



$$\frac{2\sqrt{4 - x^2}}{\sqrt{5}}$$

$$V = \int_{-2}^2 4 - x^2 dx = \frac{32}{3}$$

3.)



a.)



$$1 - y^{1/3}$$

$$V = \int_0^1 (1 - y^{1/3})^2 dy = \frac{1}{10}$$

b.)



$$V = \frac{1}{2}\pi \int_0^1 (1 - y^{1/3})^2 dy = \frac{\pi}{80}$$

c.)



$$V = \frac{\sqrt{3}}{4} \int_0^1 (1 - y^{1/3})^2 dy = \frac{\sqrt{3}}{40}$$

d.)



$$V = \frac{1}{2}\pi \int_0^1 (1 - y^{1/3})^2 dy = \frac{\pi}{40}$$

# Area and Volume Solutions



$$4 - x^2 = x + 2 \rightarrow x^2 + x - 2 = 0$$

$$x = -2 \quad x = 1$$

$$A = \int_{-2}^1 4 - x^2 - (x + 2) dx = \int_{-2}^1 2 - x - x^2 dx$$

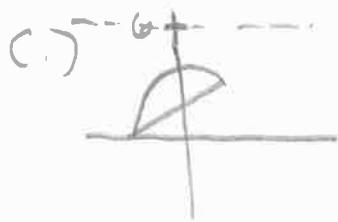
$$= \left[ 2x - \frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_{-2}^1 = (2 - \frac{1}{2} - \frac{1}{3}) - (-4 - 2 + \frac{8}{3})$$

b.)



$$4 - x^2 - (x + 2)$$

$$V = \int_{-2}^1 (4 - x^2 - (x + 2))^2 dx$$



$$R(x) = 6 - (x + 2)$$

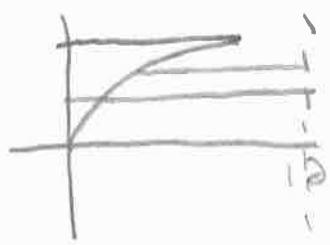
$$r(x) = 6 - (4 - x^2)$$

$$V = \pi \int_{-2}^1 (6 - (x + 2))^2 - (6 - (4 - x^2))^2 dx$$

d) Skip

2) a)  $A = \int_0^9 (6 - 2\sqrt{x}) dx = 18$

b)  $x=0 \quad x=\frac{y^2}{4} \quad x=12$



$$R(y) = 12 - 0 \quad r(y) = 12 - \frac{y^2}{4}$$

$$V = \pi \int_0^6 12^2 - (12 - \frac{y^2}{4})^2 dy$$



$$R(x) = 7 - 2\sqrt{x} \quad r(x) = 7 - x$$

$$V = \pi \int_0^9 (7 - 2\sqrt{x})^2 - 1^2 dx$$

d)

$$\boxed{\frac{3}{16}y^4}$$

$$\frac{3y^3}{4}$$

$$V = \frac{3}{16} \int_0^6 y^4 dy$$

$$\frac{y^5}{4}$$

3.) a)



$$A = \int_0^4 x^{1/2} dx = \left[ \frac{2}{3} x^{3/2} \right]_0^4 = \frac{16}{3}$$

$$\text{b.) } \int_0^r x^{1/2} dx = \frac{8}{3} \rightarrow \left[ \frac{2}{3} x^{3/2} \right]_0^r = \frac{8}{3}$$

$$\frac{2}{3} r^{3/2} = \frac{8}{3} \rightarrow r^{3/2} = 4 \rightarrow r = 4^{2/3} = \sqrt[3]{16}$$

$$\text{c.) } V = \pi \int_0^4 x dx = \pi \left[ \frac{1}{2} x^2 \right]_0^4 = 8\pi$$

$$\text{d.) } \pi \int_0^k x dx = 4\pi \rightarrow \int_0^k x dx = 4 \rightarrow \left[ \frac{1}{2} x^2 \right]_0^k = 4$$

$$\frac{k^2}{2} = 4 \rightarrow k^2 = 8 \rightarrow k = \sqrt{8}$$

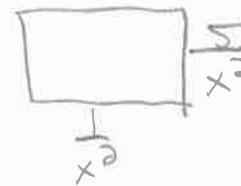
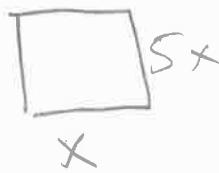
4.) a)



$$x = \frac{1}{x^2} \text{ when } x = 1$$

$$A = \int_0^1 x dx + \int_1^3 \frac{1}{x^2} dx = \frac{1}{2} + \left[ -\frac{1}{x} \right]_1^3 \\ = \frac{1}{2} + \left[ -\frac{1}{3} + 1 \right] = \frac{7}{6}$$

b.)



$$V = \int_0^1 5x^2 dx + \int_1^3 \frac{5}{x^4} dx \\ = \frac{265}{81}$$

$$\text{c.) } V = \pi \int_0^1 2^2 - (2-x)^2 dx + \pi \int_1^3 2^2 - (2-\frac{1}{x^2})^2 dx$$