CALCULUS BC
WORKSHEET 1 ON DIFFERENTIAL EQUATIONS

Work the following on **notebook paper**. Do not use your calculator.

Solve for y as a function of x.

1. $\frac{dy}{dx} = \frac{x-3}{y}$ and $y(2) = -5$

2. $y' = 2x\sqrt{y}$ and $y(2) = 25$

3. $\frac{dy}{dx} = 4y^2 \sec^2(2x)$ and $y\left(\frac{\pi}{8}\right) = 1$

4. $xy\frac{dy}{dx} = \ln x$ and $y(1) = 2$

5. $y' = 2x \sec y$ and $y(2) = -\frac{\pi}{2}$

6. $y' - xe^y = 2e^y$ and $y(0) = 0$

7. $\frac{dy}{dx} = 2xy^3 \sin\left(x^2\right)$ and $y(0) = -1$

8. $\frac{dy}{dx} = \frac{1}{y^2}$ and $y(0) = 4$

9. Find a curve in the xy-plane that passes through the point (0, 3) and whose tangent line at a point (x, y) has slope $\frac{2x}{y^2}$.

CALCULUS BC
WORKSHEET ON EULER'S METHOD

Work the following on notebook paper, showing all steps.

1. (a) Given the differential equation $\frac{dy}{dx} = x + 2$ and $y(0) = 3$. Find an approximation for $y(1)$ by using
Euler's method with two equal steps. Sketch your solution.

(b) Solve the differential equation $\frac{dy}{dx} = x + 2$ with the initial condition $y(0) = 3$, and use your solution to
find $y(1)$.

(c) The error in using Euler's Method is the difference between the approximate value and the exact value.
What was the error in your answer? How could you produce a smaller error using Euler's Method?

2. Suppose a continuous function f and its derivative f' have values that are given in the following table.
Given that $f(2) = 5$, use Euler's Method with two steps of size $\Delta x = 0.5$ to approximate the value of $f(3)$.

x	2.0	2.5	3.0
$f'(x)$	0.4	0.6	0.8
$f(x)$	5		

3. Given the differential equation $\frac{dy}{dx} = \frac{1}{x+2}$ and $y(0) = 1$. Find an approximation of $y(1)$ using Euler's
Method with two steps and step size $\Delta x = 0.5$.

4. Given the differential equation $\frac{dy}{dx} = x + y$ and $y(1) = 3$. Find an approximation of $y(2)$ using
Euler's Method with two equal steps.

5. The curve passing through (2, 0) satisfies the differential equation $\frac{dy}{dx} = 4x + y$. Find an approximation
to $y(3)$ using Euler's Method with two equal steps.

6. Assume that f and f' have the values given in the table. Use Euler's Method with two equal steps to approximate the value of $f(4.4)$.

x	4	4.2	4.4
$f'(x)$	-0.5	-0.3	-0.1
$f(x)$	2		

7. The table gives selected values for the derivative of a function f on the interval $-2 \le x \le 2$. If $f(-2) = 3$ and Euler's method with a step-size of 1.5 is used to approximate $f(1)$, what is the resulting approximation?

x	$f'(x)$
-2	-0.8
-1.5	-0.5
-1	-0.2
-0.5	0.4
0	0.9
0.5	1.6
1	2.2
1.5	3
2	3.7

8. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = x + 2y$ with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.6)$.

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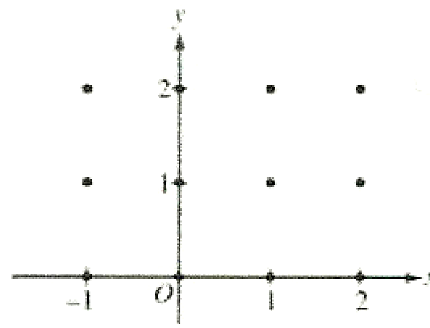
9. (2005 BC 4)

Consider the differential equation $\frac{dy}{dx} = 2x - y.$



(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and sketch the solution curve that passes through the point (0, 1).

(b) The solution curve that passes through the point (0, 1) has a local minimum at $x = \ln\left(\frac{3}{2}\right)$. What is the y-coordinate of this local minimum?

(c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$.

Show the work that leads to your answer.

(d) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.

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10. (Modified version of 2009 BC 4)

Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2 y$. Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(-1) = 2$..

(a) Use Euler's method with two steps of equal size, starting at $x = -1$, to approximate $f(0)$. Show the work that leads to your answer.

(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 2.$