CALCULUS
WORKSHEET ON INTEGRATION WITH DATA

Work the following on **notebook paper**. Give decimal answers correct to **three** decimal places.

1. A tank contains 120 gallons of oil at time $t = 0$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in gallons per hour and t is measured in hours. Selected values of $R(t)$ are given in the table below.

t (hours)	0	3	5	9	12
$R(t)$ (gallons per hour)	8.9	6.8	6.4	5.9	5.7

(a) Estimate the number of gallons of oil in the tank at $t = 12$ hours by using a trapezoidal approximation with four subintervals and values from the table. Show the computations that lead to your answer.

(b) A model for the rate at which oil is being pumped into the tank is given by the function $G(t) = 3 + \dfrac{10}{1+\ln(t+2)}$, where $G(t)$ is is measured in gallons per hour and t is measured in hours. Use the model to find the number of gallons of oil in the tank at $t = 12$ hours.

2. A hot cup of coffee is taken into a classroom and set on a desk to cool. The table shows the rate $R(t)$ at which temperature of the coffee is dropping at various times over an eight minute period, where $R(t)$ is measured in degrees Fahrenheit per minute and t is measured in minutes. When $t = 0$, the temperature of the coffee is 113° F.

t (minutes)	0	3	5	8
$R(t)$ (° F/min.)	5.5	2.7	1.6	0.8

(a) Estimate the temperature of the coffee at $t = 8$ minutes by using a left Riemann sum with three subintervals and values from the table. Show the computations that lead to your answer.

(b) Use values from the table to estimate the average rate of change of $R(t)$ over the eight minute period. Show the computations that lead to your answer.

(c) A model for the rate at which the temperature of the coffee is dropping is given by the function $y(t) = 7e^{-0.3t}$, where $y(t)$ is measured in degrees Fahrenheit per minute and t is measured in minutes. Use the model to find the temperature of the coffee at $t = 8$ minutes.

(d) Use the model given in (b) to find the average rate at which the temperature of the coffee is dropping over the eight minute period.

3. (Modification of 2001 AB 2/ BC 2)
The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table below shows the water temperature as recorded every 3 days over a 15-day period.

t (days)	0	3	6	9	12	15
$W(t)$ (°C)	20	31	28	24	22	21

(a) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days and values from the table. Show the computations that lead to your answer.

(b) A student proposes the function P, given by $P(t) = 20 + 10te^{\left(-t/3\right)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Use the function P to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

4. (Modification of 2004 Form B AB 3/ BC 3)

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le v(t) \le 40$ are shown in the table below.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7. 0	9.2	9.5	7. 0	4.5	2.4	2.4	4.3	7.2

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the average velocity of the plane, in miles per minute over the time interval $0 \le v(t) \le 40$?

5. (Modification of 2005 AB 3/ BC 3)

A metal wire of length 8 centimeters is heated at one end. The table below gives selected values of the temperature $T(x)$, in degrees Celsius, of the wire x cm from the heated end.

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

6. (Modification of 2006 AB 4/ BC 4)

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table below.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (ft per second)	5	14	22	29	35	40	44	47	49

(a) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(b) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at $t = 80$ seconds? Explain your answer.

CALCULUS
WORKSHEET ON SECOND FUNDAMENTAL THEOREM AND FUNCTIONS DEFINED BY INTEGRALS

1. Evaluate.

(a) $\dfrac{d}{dx}\displaystyle\int_3^x \frac{\sin t}{t}\,dt$ (b) $\dfrac{d}{dx}\displaystyle\int_\pi^x e^{-t^2}\,dt$ (c) $\dfrac{d}{dx}\displaystyle\int_1^{\cos x} \frac{1}{t}\,dt$

(d) $\dfrac{d}{dx}\displaystyle\int_x^2 \ln\left(t^2\right)dt$ (e) $\dfrac{d}{dx}\displaystyle\int_{-5}^{x^2} \cos\left(t^3\right)dt$ (f) $\dfrac{d}{dx}\displaystyle\int_{\tan x}^{17} \sin\left(t^4\right)dt$

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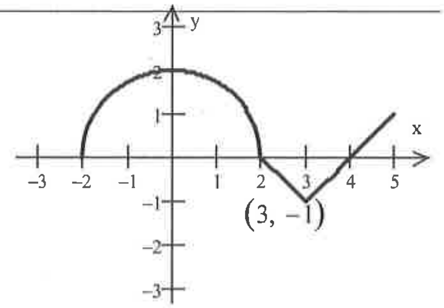
2. The graph of a function f consists of a semicircle and two line segments as shown. Let g be the function given by $g(x)=\int_0^x f(t)\,dt$.



Graph of f

(a) Find $g(0), g(3), g(-2)$, and $g(5)$.
(b) Find all values of x on the open interval $(-2,5)$ at which g has a relative maximum. Justify your answers.
(c) Find the absolute minimum value of g on the closed interval $[-2,5]$ and the value of x at which it occurs. Justify your answer.
(d) Write an equation for the line tangent to the graph of g at $x=3$.
(e) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-2,5)$. Justify your answer.
(f) Find the range of g.

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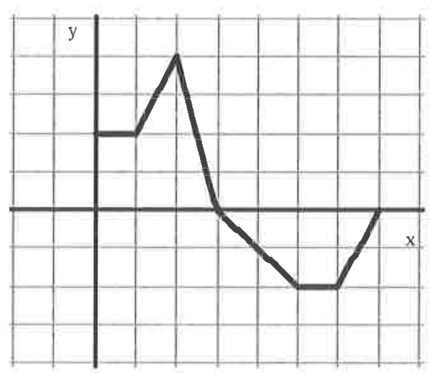
3. Let $g(x)=\int_0^x f(t)\,dt$, where f is the function whose graph is shown.



Graph of f

(a) Evaluate $g(0), g(1), g(2)$, and $g(7)$.
(b) Write an equation for the line tangent to the graph of g at $x=4$.
(c) On what intervals is g increasing? Decreasing? Justify your answer.
(d) Find all values of x on the open interval $0<x<7$ at which g has a relative maximum. Justify your answer.
(e) Where does g have its absolute maximum value? What is the maximum value? Justify your answer.
(f) Where does g have its absolute minimum value? What is the minimum value? Justify your answer.

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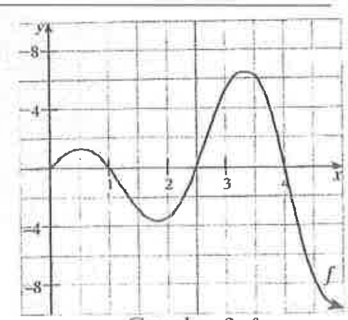
4. Let $g(x)=\int_0^x f(t)\,dt$, where f is the function whose graph is shown.



Graph of f

(a) On what intervals is g decreasing? Justify.
(b) For what value(s) of x does g have a relative maximum? Justify.
(c) On what intervals is g concave down? Justify.
(d) At what values of x does g have an inflection point? Justify.

TURN->>>

CALCULUS BC
WORKSHEET ON 5.2

Work the following on **notebook** **paper**. Do not use your calculator.

1. $\int \frac{1}{2x+5}\,dx$
2. $\int (x^3+1)^5 x^2\,dx$
3. $\int \frac{x}{x^2+4}\,dx$
4. $\int x\sin(3x^2)\,dx$
5. $\int \frac{x^2}{\sqrt{x^3+2}}\,dx$
6. $\int \tan^5 x\sec^2 x\,dx$
7. $\int \frac{\cos x}{\sqrt{\sin x}}\,dx$
8. $\int \frac{\sin x}{1+\cos x}\,dx$
9. $\int_0^1 x(x^2+1)^3\,dx$
10. $\int_0^4 \frac{2x}{\sqrt{x^2+9}}\,dx$
11. $\int_0^2 x\sqrt[3]{4+x^2}\,dx$
12. $\int_1^2 \frac{x-2}{x}\,dx$
13. $\int_0^{\pi/2} \sin^3 x\cos x\,dx$
14. $\int_1^{e^3} \frac{\ln x}{x}\,dx$
15. $\int_0^2 \frac{x^2-2}{x+1}\,dx$
16. $\int_0^{\pi/2} \cos\left(\frac{2x}{3}\right)dx$
17. $\int_2^6 \frac{x-4}{x+1}\,dx$
18. $\int_{e^2}^{e^3} \frac{1}{x\ln x}\,dx$

CALCULUS BC
WORKSHEET ON 5.2

Work the following on **notebook** **paper**. Do not use your calculator.

1. $\int \frac{1}{2x+5}\,dx$
2. $\int (x^3+1)^5 x^2\,dx$
3. $\int \frac{x}{x^2+4}\,dx$
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5. $\int \frac{x^2}{\sqrt{x^3+2}}\,dx$
6. $\int \tan^5 x\sec^2 x\,dx$
7. $\int \frac{\cos x}{\sqrt{\sin x}}\,dx$
8. $\int \frac{\sin x}{1+\cos x}\,dx$
9. $\int_0^1 x(x^2+1)^3\,dx$
10. $\int_0^4 \frac{2x}{\sqrt{x^2+9}}\,dx$
11. $\int_0^2 x\sqrt[3]{4+x^2}\,dx$
12. $\int_1^2 \frac{x-2}{x}\,dx$
13. $\int_0^{\pi/2} \sin^3 x\cos x\,dx$
14. $\int_1^{e^3} \frac{\ln x}{x}\,dx$
15. $\int_0^2 \frac{x^2-2}{x+1}\,dx$
16. $\int_0^{\pi/2} \cos\left(\frac{2x}{3}\right)dx$
17. $\int_2^6 \frac{x-4}{x+1}\,dx$
18. $\int_{e^2}^{e^3} \frac{1}{x\ln x}\,dx$

CALCULUS
WORKSHEET ON INVERSES

Work the following on **notebook paper**. **No calculator**.
On problems 1 – 3,
(a) Find the inverse function of *f*.
(b) Graph f and f^{-1} on the same set of coordinate axes.
(c) Describe the relationship between the graphs.
(d) State the domain and range of f and f^{-1}.

1. $f(x) = 2x - 3$
2. $f(x) = \sqrt{x-2}$
3. $f(x) = x^3 + 1$

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4. Let $g(x) = x^5 + 3x - 2$. Find $\left(g^{-1}\right)'(2)$.

5. Let $f(x) = \frac{1}{4}x^3 + x - 1$. Find $\left(f^{-1}\right)'(3)$.

6. Let $f(x) = 3x^4 + x$, and let g be the inverse function of *f*. Find the value of $g'(2)$.

7. Let $h(x) = \sqrt{x-4}$. Find $\left(h^{-1}\right)'(3)$.

8. Let $g(x) = \sqrt[3]{x}$. Find $\left(g^{-1}\right)'(2)$.

9. Let $f(x) = \sin x,\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$, and let g be the inverse of f. Find $g'\left(\frac{1}{2}\right)$.

10. Let $f(x) = \tan x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$. Find $\left(f^{-1}\right)'\left(\sqrt{3}\right)$.

11. If $f(4) = 5$ and $f'(4) = \frac{2}{3}$, find $\left(f^{-1}\right)'(5)$.

12. If $g(7) = 3$ and $g'(3) = \frac{5}{6}$ and $g'(7) = \frac{3}{4}$, find $\left(g^{-1}\right)'(3)$.

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13. The following table shows values of a differentiable function f and its derivative.

x	f	f'
1	2	½
2	3	1
3	4	2
4	6	4

If $h(x) = f^{-1}(x)$, find $h'(3)$.