CALCULUS BC

SECTION I, Part B

Time—50 minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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76. Let f be a function whose derivative is given by $f'(x) = \ln\left(x^4 + 5x^3 + x^2 - 7x + 28\right)$. On the open interval $(-4, 1)$, at which of the following values of x does f attain a relative maximum?

(A) -3.623 only

(B) -0.871 only

(C) -3.623 and -3.284

(D) -3.459 and 0.581 only

(E) -3.459, -0.871, and 0.581

a	$\lim_{x \to a^-} f(x)$	$\lim_{x \to a^+} f(x)$	$f(a)$
-1	4	6	4
0	-3	-3	5
1	2	2	2

77. The function f has the properties indicated in the table above. Which of the following must be true?

(A) f is continuous at $x = -1$

(B) f is continuous at $x = 0$

(C) f is continuous at $x = 1$

(D) f is differentiable at $x = 0$

(E) f is differentiable at $x = 1$.

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78. What is the area of the region in the first quadrant enclosed by the graphs of $y = \sin(2x)$ and $y = x$

(A) 0.208 (B) 0.210 (C) 0.266 (D) 0.660 (E) 0.835

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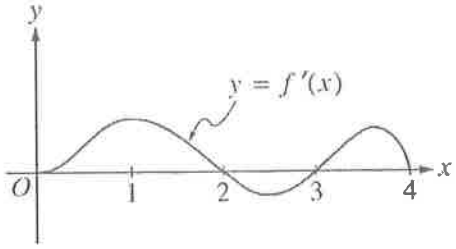
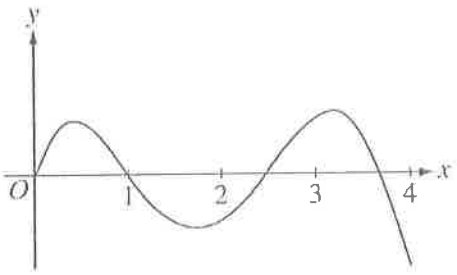


79. The figure above shows the graph of f', the derivative of the function f. If $f(0) = 0$, which of the following could be the graph of f?

(A)



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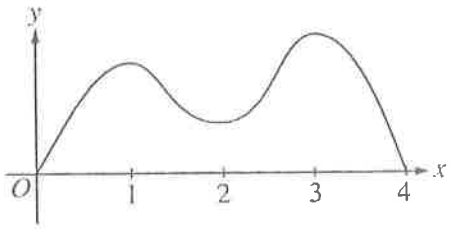

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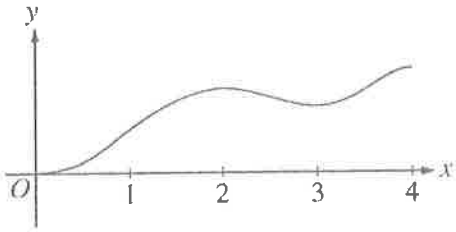

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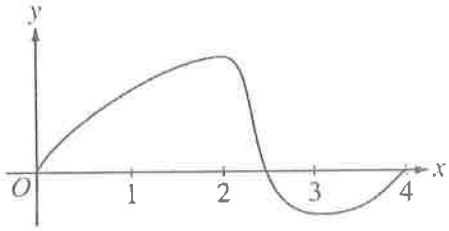
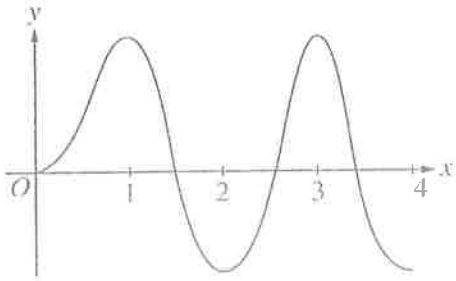


(E)



80. The volume of a certain cone for which the sum of its radius, r, and height is constant is given by $V = \frac{1}{3}\pi r^2(10 - r)$. The rate of change of the radius of the cone with respect to time is 6. In terms of r, what is the rate of change of the volume of the cone with respect to time?

(A) $-24\pi r$ (B) $6\pi r$ (C) $\frac{20}{3}\pi r - \pi r^2$ (D) $16\pi r - \frac{4}{3}\pi r^2$ (E) $40\pi r - 6\pi r^2$

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81. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit (°F). If the initial temperature of the tea, at time $t = 0$ minutes, is 200°F and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

(A) 175°F (B) 130°F (C) 95°F (D) 70°F (E) 45°F

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82. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \geq 1$. If $\sum_{n=1}^{\infty} a_n$ converges, which of the following must be true?

(A) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(C) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(D) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(E) If $b_n \leq a_n$, then the behavior of $\sum_{n=1}^{\infty} b_n$ cannot be determined from the information given.

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0	4	10	18	28	40	54

83. The table above gives selected values for a continuous function f. If f is increasing over the closed interval $[0,3]$, which of the following could be the value of $\int_0^3 f(x)dx$?

(A) 50 (B) 62 (C) 77 (D) 100 (E) 154

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84. Let f be a function with derivative given by $f'(x) = x^3 - 5x^2 + e^x$. On which of the following intervals is the graph of f concave down?

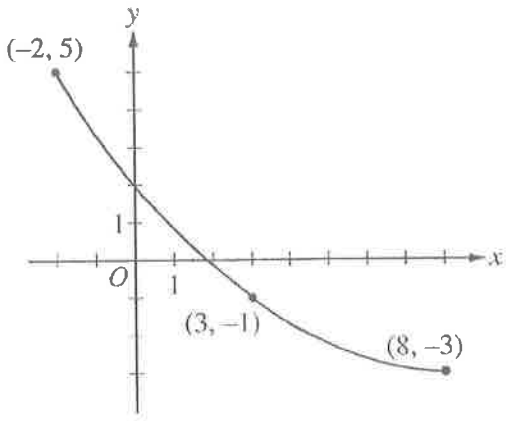
(A) $(-\infty, 0.117)$ only

(B) $(-\infty, 1.144)$

(C) $(0.116, 2.062)$

(D) $(0.673, 2.863)$

(E) $(2.863, \infty)$



Graph of f

85. A portion of the graph of a differentiable function f is shown above. If the value $c = 3$ satisfies the conclusion of the Mean Value Theorem applied to f on the open interval $-2 < x < 8$, what is the slope of the line tangent to the graph of f at $x = 3$?

(A) $-\frac{7}{5}$ (B) $-\frac{5}{4}$ (C) $-\frac{4}{5}$ (D) $-\frac{5}{7}$ (E) $-\frac{1}{5}$

86. If $f'(x) > 0$ for all x and $f''(x) < 0$ for all x, which of the following could be a table of values for f?

(A)

x	$f(x)$
-1	4
0	3
1	1

(B)

x	$f(x)$
-1	4
0	4
1	4

(C)

x	$f(x)$
-1	4
0	5
1	6

(D)

x	$f(x)$
-1	4
0	5
1	7

(E)

x	$f(x)$
-1	4
0	6
1	7

87. The position of a particle moving in the xy-plane is given by the parametric functions $x(t)$ and $y(t)$ for which $x'(t) = t \sin t$ and $y'(t) = 5e^{-3t} + 2$. What is the slope of the line tangent to the path of the particle at the point at which $t = 2$?

(A) 0.904 (B) 1.107 (C) 1.819 (D) 2.012 (E) 3.660

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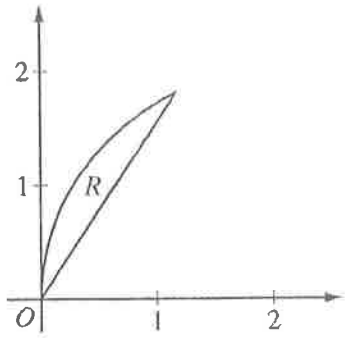

88. Let R be the region in the first quadrant that is bounded above by the polar curve $r = 4\cos\theta$ and below by the line $\theta = 1$, as shown in the figure above. What is the area of R?

(A) 0.317 (B) 0.465 (C) 0.929 (D) 2.618 (E) 5.819

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89. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y-2}$ and the lines $x = 0$ and $y = 5$ is revolved about the y-axis?

(A) 3.464 (B) 4.500 (C) 7.854 (D) 10.883 (E) 14.137

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90. Which of the following statements are true about the series $\sum_{n=2}^{\infty} a_n$, where $a_n = \dfrac{(-1)^n}{\sqrt{n} + (-1)^n}$?

I. The series is alternating.

II. $|a_{n+1}| \leq |a_n|$ for all $n \geq 2$

III. $\lim_{n \to \infty} a_n = 0$

(A) None

(B) I only

(C) I and II only

(D) I and III only

(E) I, II, and III

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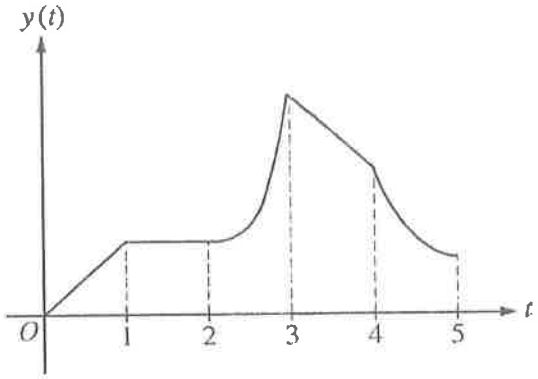

91. A particle moves along the y-axis. The graph of the particle's position $y(t)$ at time t is shown above for $0 \le t \le 5$. For what values of t is the velocity of the particle negative and the acceleration positive?

(A) $0 < t < 1$ (B) $1 < t < 2$ (C) $2 < t < 3$ (D) $3 < t < 4$ (E) $4 < t < 5$

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92. If f is a function such that $f'(x) = -f(x)$, then $\int x f(x)\,dx =$

(A) $f(x)(x+1) + C$

(B) $-f(x)(x+1) + C$

(C) $\frac{x^2}{2} f(x) + C$

(D) $-\frac{x^2}{2} f(x) + C$

(E) $-\frac{x^2}{2} f(x)\left(1 + \frac{x}{3}\right) + C$

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