CALCULUS BC
WORKSHEET ON ALTERNATING SERIES AND REMAINDERS

Work these on **notebook paper**. Use your calculator on problems 1 – 5, and give decimal answers correct to three decimal places.

1. Approximate the sum, S, of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ by using its first five terms, and explain why your estimate differs from the actual value by less than .009. Then use your results to find an interval in which S must lie.

2. Approximate the sum, S, of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}3}{n^2}$ by using its first six terms, and explain why your estimate differs from the actual value by less than .07. Then use your results to find an interval in which S must lie.

3. Approximate the sum of the convergent series $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!}$ so that the error will be less than $\frac{1}{1000}$.

   How many terms were needed? What are the properties of the terms of this series that guarantee that your approximation is within $\frac{1}{1000}$ of the exact value? Justify your answer.

4. Approximate the sum of the convergent series $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$ so that the error will be less than $\frac{1}{1000}$.

   How many terms were needed? What are the properties of the terms of this series that guarantee that your approximation is within $\frac{1}{1200}$ of the exact value? Justify your answer.

5. Approximate the sum of the convergent series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ so that the error will be less than $\frac{1}{1000}$.

   How many terms were needed? What are the properties of the terms of this series that guarantee that your approximation is within $\frac{1}{1000}$ of the exact value? Justify your answer.

Determine whether each of the given **series** converges or diverges. Justify your answer.

6. $\sum_{n=1}^{\infty} \frac{10}{n^{3/2}}$

7. $\sum_{n=1}^{\infty} \frac{1}{n^2+5}$

8. $\sum_{n=1}^{\infty} \frac{3^n}{n^2}$

9. $\sum_{n=1}^{\infty} \frac{1}{2^n+1}$

10. $\sum_{n=1}^{\infty} \frac{4}{3^n}$

11. $\sum_{n=1}^{\infty} \left(\frac{4}{3}\right)^n$

12. $\sum_{n=1}^{\infty} \frac{3n^2}{2n^2+1}$

13. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

1.) $1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} = \frac{3}{8}$

The series is alternating in sign with terms decreasing to 0 so the $|\text{error}| < \frac{1}{5!}$ which is the 1st omitted term.

$\frac{1}{120} = .008333 < .009.$

$$\frac{11}{30} < S < \frac{23}{60}$$

2.) $3 - \frac{3}{4} + \frac{3}{9} - \frac{3}{16} + \frac{3}{25} - \frac{3}{36} = \frac{973}{400} = 2.4325$

The series is alternating in sign with terms decreasing to 0 so the $|\text{error}| < \frac{3}{49}$ which is the 1st omitted term.

$\frac{3}{49} = .061224 < .07.$

$$2.37128 < S < 2.49372$$

3.) $\frac{1}{2^n n!} < \frac{1}{1000}$. Trial and error.

If $n=4$, the term is $\frac{1}{384}$. If $n=5$ the term is $\frac{1}{3840}$. We will use the 1st 5 terms (notice $n=0$ is the 1st term) to approximate the sum.

$S \approx 1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{48} + \frac{1}{384} = \frac{233}{384} = .606771$

The series is alternating in sign with terms decreasing to 0 so the $|\text{error}| < \frac{1}{3840}$ which is the 1st omitted term.

$\frac{1}{3840} < \frac{1}{1000}.$

4.) $\frac{1}{(2n)!} < \frac{1}{1000}$. If $n=3$, the term is $\frac{1}{720}$.

If $n=4$, the term is $\frac{1}{40320}$. we need 4 terms

$S \approx 1 - \frac{1}{2} + \frac{1}{24} - \frac{1}{720} = \frac{389}{720} = .540278.$

The series is alternating in sign with terms decreasing to 0. The $|\text{error}| < \frac{1}{40320}$ which is the 1st omitted term. $\frac{1}{40320} < \frac{1}{1000}$.

5.) $\frac{1}{n^3} < \frac{1}{1000} \rightarrow n^3 > 1000 \rightarrow n > 10$

we need 10 terms (notice $n=1$).

$S \approx 1 - \frac{1}{8} + \frac{1}{27} - \frac{1}{64} + \frac{1}{125} - \frac{1}{216} + \frac{1}{343} - \frac{1}{512} + \frac{1}{729} - \frac{1}{1000}$

$S \approx .901116$. The series is alternating in sign with terms decreasing to 0.

$|\text{error}| < \frac{1}{1331}$ which is the 1st omitted term.

6.) $\sum_{n=1}^{\infty} \frac{10}{n^{3/2}} = 10 \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ = Convergent, p-series.

7.) Convergent. Direct Comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

8.) Divergent. n^{th} term test.

9.) Convergent. Direct Comparison to $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$

10.) Convergent. Geometric series with $r = 1/3$.

11.) Divergent. Geometric series with $r = 4/3$

12.) Divergent. N^{th} term test. $\lim_{n\to\infty} \frac{3n^2}{2n^2+1} = \frac{3}{2} \neq 0$

13.) Divergent. Direct Comparison to $\sum_{n=1}^{\infty} \frac{1}{n}$.