NATIONAL MATH + SCIENCE INITIATIVE

AP Calculus

Calculator Skills for the AP Exam

Presenter Notes

2016-2017 EDITION

Student Study Session - Presenter Notes

Thank you for agreeing to present at one of NMSI's Saturday Study Sessions. We are grateful you are sharing your time and expertise with our students. Saturday mornings can be a "tough sell" for students, so we encourage you to incorporate strategies and techniques to encourage student movement and engagement. Suggestions for different presentation options are included in this document. If you have any questions about the content or about presentation strategies, please contact Mathematics Director Charla Holzbog at cholzbog@nms.org or AP Calculus Content Specialist Karen Miksch at kmiksch@nms.org.

The material provided contains many released AP multiple choice and free response questions as well as some AP-like questions that we have created. The goal for the session is to let the students experience a variety of both types of questions to gain insight on how the topic will be presented on the AP exam. It is also beneficial for the students to hear a voice other than their teacher in order to help clarify their understanding of the concepts.

Suggestions for presenting:
The vast majority of the study sessions are on Saturday and students and teachers are coming to be WOWed! We want activities to engage the students as well as prepare them for the AP Exam. The following presenter notes include pacing suggestions (you only have 50 minutes!), solutions, and recommended engagement strategies.

Suggestions on how to prepare:

- The notes/summaries on the last page(s) are for reference. We want the students' time during the session to be focused on the questions as much as possible and not taking or reading the notes. As the questions are presented during the session, you may wish to refer the students back to those pages as needed. It is not our intent for the sessions to begin with a lecture over these pages.
- As you prepare, work through the questions in the packet noting the level of difficulty and topic or skill required for the questions.
- Design a plan for what questions you would like to cover with the group depending on their level of expertise. Some groups will be ready for the tougher questions while other groups will need more guidance and practice on the easier ones. Create an easy, medium, and hard listing of the questions prior to the session. This will allow you to adjust on the fly as you get to know the groups. In most instances, there will **not** be enough time to cover all the questions in the packet. Use your judgement on the amount of questions to cover based on the students' interactions. Remember to include both multiple choice and free response type questions. Discussions on test taking strategies and scoring of the free response questions are always great to include during the day.
- The concepts should have been previously taught; however, be prepared to "teach" the topic if you find out the students have not covered the concept prior in class. In sessions where multiple schools come together, you might have a mixture of students with and without prior knowledge on the topic. You will have to use your best judgement in this situation.
- Consider working through some free response questions before the multiple choice questions, or flipping back and forth between the two types of questions. Sometimes, if free response questions are saved for the last part of the session, it is possible students only get practice with one or two of them and most students need additional practice with free response questions.

Calculus Skills for the AP Exam Presenter Notes and Solutions

This session includes a reference sheet at the back of the packet. We suggest that the presenter spends a few minutes only on the calculator skills requirements and does not spend time going over the whole reference sheet, but may point out to students that it is available to refer to if needed. We suggest that students will work in pairs (depending on the size of the class), arrange the desks prior to the start of the session.

We have intentionally included more material than can be covered in most Student Study Sessions to account for groups that are able to answer the questions at a faster rate. Use our presenter notes and your own judgment, based on the group of students, to determine the order and selection of questions to work in the session. Be sure to include a variety of types of questions (multiple choice, free response, calculator, and non-calculator) in the time allotted. Notice in the solutions guide the questions are categorized as 3, 4, or 5 indicating a typical question of the difficulty level (DL) for a student earning these qualifying scores on the AP exam.

I. 10 Minutes Introductory Activity

- When students enter the room, ask them to review the reference sheet about the requirements for calculator use on the AP Exam in the back of their packet as they wait a few minutes for the start of the session. Encourage them to highlight or underline key points. Discuss key points, especially the exam format as well as the bold bullets.
- Instruct students to do #15b. Walk around and monitor students, listening to discussions to determine the level of the group.

II. 20 minutes Free Response Practice

- Model the remainder of FR #15. Focus on the set-ups for each part of the question and encourage students to use the function names in their set-ups, remind them not to do any preliminary rounding, store decimal values, and use "NUT" in their justifications- **n**oun, **u**nit, **t**ime to earn full credit.
- Let the students to work on problems 13 or 14 in groups of 4 (combine 2 sets of pairs for this). You may choose to assign one of the questions to entire group or split the two questions and then have a representative group go over their solutions. Discuss the rubric(s) and how to earn the most points for their work.

III. 20 minutes Multiple Choice Practice

- Remind students that even though there are 15 questions on the calculator portion of the multiple choice on the AP exam, only 6 or 7 require the actual use of a calculator. Problem 1 and part of #11 can be done without the use of a calculator. Suggest students graph #11 to see why III is true but not to do any actual integration in this problem, then try #1 without a calculator. We have chosen to focus this session on the types of questions that would require the use of a calculator.
- Notice that these questions provide a nice review of topics covered throughout the year for derivatives and integrals. After putting students back in pairs, play speed dating for a variety of questions for the remainder of the time. Suggested order and questions: 2, 9, 7, 8, 6, 3, 12, 5, and 4 as time allows.

Multiple Choice Solutions

(Note: When justifying answers, do not write calculator syntax on paper. Calculus notation must be used on the free response section of the AP exam in order to receive credit for the solution. Setting an appropriate window is very important--use the window given in the problem or base it on the answer choices.)

1. B. (2012 AB79) DL: 4
   Use the average value theorem for integrals to determine the average velocity of the particle is given by $\frac{1}{8}\int_0^8 v(t)\,dt$. For the interval $[0, 8]$, let $x(t)$ represent position of the particle and $\int_0^8 v(t)\,dt = x(8) - x(0)$ is displacement so the average velocity is $\frac{displacement}{change\ in\ time} = \frac{1}{8}\int_0^8 v(t)\,dt$.

2. C (AP-like) DL: 3
   Graph $f'(x)$ and note where its slope is negative (in other words, $f'(x)$ is decreasing) for $1 < x < 6$. $f''(x) < 0$ on the interval $(3.105, 6)$.

3. D. (2012 AB81) DL: 4
   Use the initial condition and determine that the amount of water in the tank after 20 minutes $= 800 + \int_0^{20} 30(1 - e^{-.16t})\,dt = 1220$ gallons.

4. B (AP-like) DL: 3
   The velocity of x_1 is $\frac{2}{t}$, the acceleration of x_1 is $\frac{-2}{t^2}$. The velocity of x_2 is $2t - 10$. The point of intersection for $\frac{-2}{t^2} = 2t - 10$ has $t \approx 0.470$.

5. A (AP-like) DL: 3
   The average value is found using $\frac{1}{2}\int_0^2 \cos(x^2)\,dx \approx 0.231$

6. B. (2012 AB89) DL: 4
   Determine an equation relating the variables and solves the related rates application:
   Let $x =$ distance from pole to man, $z =$ length of streetlight shadow.
   $\frac{15}{x+z} = \frac{6}{z}; 15z = 6(x+z); 15z = 6x + 6z;\ 9z = 6x;\ 9\frac{dz}{dt} = 6\frac{dx}{dt};\ 9\frac{dz}{dt} = 6(4); \frac{dz}{dt} = \frac{24}{9} = 2.667$
   Therefore the person's shadow is lengthening at a rate of 2.667 ft./sec.

7. B (AP-like) DL: 4

$f'(x) = \dfrac{1}{\sqrt{1-x^2}}$

By the Mean Value Theorem, $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$f'(c) = \dfrac{1}{\sqrt{1-c^2}} = \dfrac{\frac{\pi}{2}-0}{1-0}$

Solve for c to determine $c \approx 0.771$.

8. E. (2012 AB89) DL: 4

Use the Fundamental Theorem of Calculus:

$v(3) = 5 + \int_0^3 \dfrac{t+3}{\sqrt{t^3+1}}\,dt = 11.710$

9. B (AP-like) DL: 3

$f'(x) = \dfrac{x^2}{(e^x+1)(\sin x)}$

$\dfrac{d}{dx}\left(\dfrac{x^2}{(e^x+1)(\sin x)}\right)_{x=1} \approx 0.200$

10. B (AP-like) DL: 3

Determine the upper bound of the integral for area by finding the $x-$value of the point of intersection of $y = e^{-x}$ and $y = \ln x$. Then use the calculator to integrate $\int_1^{1.3097996} (e^{-x} - \ln x)\,dx \approx 0.054$. (upper bound may be stored into the calculator as a letter)

11. D (AP-like) DL: 4

$g'(x) = e^x(x^2-1)$. Graph the integrand to see that since the integrand has more area above the $x-$axis than below, $g(2) = \int_0^2 e^x(x^2-1)dx > 0$. Do not use the calculator to evaluate $g(2)$ since using the calculator unnecessarily uses too much time on a timed test.

12. D (AP-like) DL: 4

Use the washer method after finding the point of intersection of the two graphs in the first quadrant and evaluate $\pi\int_0^{2.031}\left((x+16)^2-(x^4+1)^2\right)dx \approx 1594.480$.

Free Response Solutions

13. (2012 AB/BC 1) (Numerical)

(a) $W'(12) \approx \dfrac{W(15)-W(9)}{15-9} = \dfrac{67.9-61.8}{6}$
$= 1.017$ (or 1.016)

The water temperature is increasing at a rate of approximately 1.017 °F per minute at time $t = 12$ minutes.

2 { 1: estimate; 1: interpretation with units }

(b) $\displaystyle\int_0^{20} W'(t)\,dt = W(20) - W(0) = 71.0 - 55.0 = 16$

The water has warmed by 16 °F over the interval from $t = 0$ to $t = 20$ minutes.

2 { 1: value; 1: interpretation with units }

(c) $\dfrac{1}{20}\displaystyle\int_0^{20} W(t)\,dt \approx \frac{1}{20}(4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15))$
$= \dfrac{1}{20}(4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9)$
$= \dfrac{1}{20} \cdot 1215.8 = 60.79$

This approximation is an underestimate, because a left hand Riemann sum is used and the function W is strickly increasing.

3 { 1: left Riemann sum; 1: approximation; 1: underestimate with reason }

(d) $W(25) = 71.0 + \displaystyle\int_{20}^{25} W'(t)\,dt$
$= 71.0 + 2.043155 = 73.043$

2 { 1: integral; 1: answer }

14. (2015 AB/BC2) (Graphical)

(a) The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at the points (0,2), and (2,4), and $(A,B) = (1.032832, 2.401108)$.

$$\text{Area} = \int_0^A [g(x) - f(x)]dx + \int_A^2 [f(x) - g(x)]dx$$
$$= 0.997427 + 1.006919 = 2.004$$

4: 1: limits; 2: integrands; 3: answer

(b) Volume $= \int_A^2 [f(x) - g(x)]^2 dx = 1.283$

3: 2: integrand; 1: answer

(c) $h(x) = f(x) - g(x)$
$h'(x) = f'(x) - g'(x)$
$h'(1.8) = f'(1.8) - g'(1.8) = -3.812$ (or -3.811)

2: 1: considers h'; 1: answer

15. (2014 AB/BC1)-(Analytical)

(a) $\dfrac{A(30)-A(0)}{30-0}=-0.197(\text{or } -0.196)$lbs/day	1: answer with units
(b) $A'(15)=-0.164(\text{or } -0.163)$ The amount of grass clippings in the bin is decreasing at a rate of 0.164 (or 0.163) lbs/day at time $t=15$ days.	2 { 1: $A'(15)$ 1: interpretation
(c) $A(t)=\dfrac{1}{30}\int_0^{30}A(t)\,dt \Rightarrow t=12.415(\text{or } 12.414)$	2 { 1: $\dfrac{1}{30}\int_0^{30}A(t)\,dt$ 1: answer
(d) $L(t)=A(30)+A'(30)\bullet(t-30)$ $A'(30)=-0.055976$ $A(30)=0.782928$ $L(t)=0.5\Rightarrow t=35.054$	4 { 2: expression for $L(t)$ 1: $L(t)=0.5$ 1: answers