

Name: \_\_\_\_\_

## Limits Review

Find the limit of the function:

1.  $\lim_{x \rightarrow 3} \frac{x-3}{x^2-9}$   $\frac{1}{6}$
2.  $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 5x}$   $\frac{3}{5}$
3.  $\lim_{h \rightarrow 0} \frac{-5(x+h)+5x}{h}$   $-5$
4.  $\lim_{x \rightarrow 3} \begin{cases} \frac{1}{2}x+1 & x \leq 4 \\ 2x-5 & x > 4 \end{cases}$   $\frac{5}{2}$
5.  $\lim_{x \rightarrow 4} \frac{\sqrt{x+5}-3}{x-4}$   $\frac{1}{6}$
6.  $\lim_{x \rightarrow 3^-} \frac{x^2}{x^2-9}$   $-\infty$
7.  $\lim_{x \rightarrow 0} \frac{(1-\cos^2 x)}{2x^2}$   $\frac{1}{2}$
8.  $\lim_{x \rightarrow 5^+} \frac{x}{x^2-25}$   $\infty$
9.  $\lim_{h \rightarrow 0} \frac{(x+h)^2-x^2}{h}$   $2x$
10.  $\lim_{x \rightarrow 2^+} \frac{x-3}{2-x}$   $\infty$
11.  $\lim_{x \rightarrow 0} \frac{\sin x \cos x}{x^2-5x}$   $-\frac{1}{5}$
12.  $\lim_{x \rightarrow 3} \frac{\sqrt{x+1}}{x-4}$   $-2$
13.  $\lim_{x \rightarrow 0} \frac{\frac{1}{x+2} - \frac{1}{2}}{x}$   $-\frac{1}{4}$
14.  $\lim_{x \rightarrow \infty} \frac{4x-3}{2-x}$   $-4$
15.  $\lim_{x \rightarrow 5} \begin{cases} x-4 & x \leq 5 \\ -x+3 & x > 5 \end{cases}$  DNE
16.  $\lim_{x \rightarrow \infty} \frac{7x^2-2}{(2x-1)(3-x)}$   $-\frac{7}{2}$
17.  $\lim_{x \rightarrow 2^+} \begin{cases} x^2-2 & x \leq 2 \\ 3x-5 & x > 2 \end{cases}$   $1$
18.  $\lim_{x \rightarrow \infty} \frac{3x^2-7x}{2x^3}$   $0$
19.  $\lim_{x \rightarrow 0} \frac{\sin x(1-\cos x)}{2x^2}$   $0$
20.  $\lim_{x \rightarrow 1^-} \begin{cases} x^3 & x \leq 1 \\ -x+3 & x > 1 \end{cases}$   $1$

21. Use the Intermediate Value Theorem to show that  $f(x) = x^3 + 2x - 1$  has a zero on the interval  $[0,1]$

Find where the function is discontinuous and describe the type of discontinuity

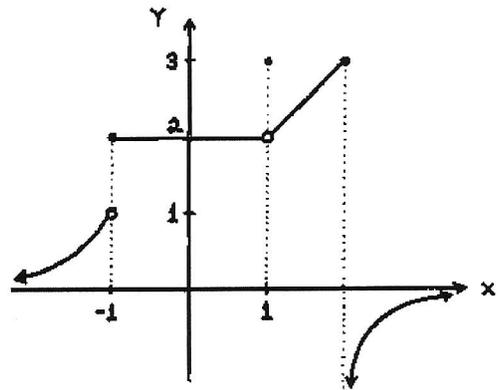
22.  $f(x) = \begin{cases} \frac{x^2-4}{x+2} & x \neq 2 \\ 3 & x = 2 \end{cases}$  RD / hole @  $x=2$
23.  $f(x) = \begin{cases} x^2 & x < 1 \\ 2x-1 & 1 \leq x \leq 3 \\ -x+7 & x > 3 \end{cases}$  break @  $x=3$
24.  $f(x) = \frac{x^2-3x-10}{x^2-25}$  RD @  $x=5$ , VA @  $x=-5$

25. Find the value of  $c$  that will make  $f(x)$  continuous at  $x=1$

$$f(x) = \begin{cases} x^2+1 & x \leq 1 \\ cx+4 & x > 1 \end{cases} \quad -2$$

Use the graph below of  $g(x)$  to answer the questions

26.  $\lim_{x \rightarrow -1^+} g(x)$   $2$
27.  $\lim_{x \rightarrow 2^+} g(x)$   $-\infty$
28.  $\lim_{x \rightarrow -1^-} g(x)$   $1$
29.  $\lim_{x \rightarrow 2^-} g(x)$   $3$
30.  $\lim_{x \rightarrow 1} g(x)$   $2$
31.  $\lim_{x \rightarrow 0} g(x)$   $2$
32.  $\lim_{x \rightarrow -1} g(x)$  DNE
33.  $g(1)$   $3$
34.  $\lim_{x \rightarrow \infty} g(x)$   $0$



35. Find where  $g(x)$  is discontinuous and list the type of discontinuity

$x = -1$  break  
 $x = 1$  RD  
 $x = 2$  VA (2,3 should be an open circle)

$$\textcircled{1} \lim_{x \rightarrow 3} \frac{x-3}{(x-3)(x+3)} = \boxed{\frac{1}{6}}$$

$$\textcircled{2} \lim_{x \rightarrow 0} \frac{3x \sin 3x}{3x-1} \cdot \lim_{x \rightarrow 0} \frac{1}{\sin 5x} \cdot \frac{5x}{5x}$$

$$\frac{3x}{5x} = \boxed{\frac{3}{5}}$$

$$\textcircled{3} \lim_{h \rightarrow 0} \frac{-5(x+h) + 5x}{h} = \frac{-5x - 5h + 5x}{h} = \boxed{-5}$$

$$\textcircled{4} \lim_{x \rightarrow 3} \begin{cases} \frac{1}{2}x + 1 & x \leq 4 \\ 2x - 5 & x > 4 \end{cases}$$

$$\frac{1}{2}(3) + 1 = \frac{5}{2}$$

$$\textcircled{5} \frac{\sqrt{x+5} - 3}{x-4} \cdot \frac{\sqrt{x+5} + 3}{\sqrt{x+5} + 3} = \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)} = \frac{x-4}{(x-4)(\sqrt{x+5}+3)} = \frac{1}{\sqrt{x+5}+3} = \boxed{\frac{1}{6}}$$

$$\textcircled{6} \lim_{x \rightarrow 3^-} \frac{x^2}{x^2-9} = \frac{3}{0} = \frac{(2.9)^2}{2.9^2-9} = -\infty$$

$$\textcircled{7} \lim_{x \rightarrow 0} \frac{\sin^2 x}{2x^2} = \frac{1}{2} \cdot \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{\sin x}{x} = \boxed{\frac{1}{2}}$$

$$\textcircled{8} \lim_{x \rightarrow 5^+} \frac{5}{25-25} = \frac{5}{0} = \frac{5 \cdot 1}{(5 \cdot 1)^2 - 25} = \infty$$

$$\textcircled{9} \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2hx + h^2 - x^2}{h} = \frac{2hx + h^2}{h} = 2x + h = \boxed{2x}$$

$$\textcircled{10} \lim_{x \rightarrow 2^+} \frac{x-3}{2-x} = \frac{-1}{0} = \frac{2.1-3}{2-2.1} = \infty$$

$$\textcircled{11} \lim_{x \rightarrow 0} \frac{\sin x \cos x}{x(x-5)} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{\cos x}{x-5} = 1 \cdot \frac{1}{-5} = \boxed{-\frac{1}{5}}$$

$$(12) \lim_{x \rightarrow 3} \frac{\sqrt{x+1}}{x-4} = \frac{2}{-1} = \boxed{-2}$$

$$(13) \lim_{x \rightarrow 0} \frac{\frac{1}{x+2} - \frac{1}{2} - \frac{1}{2} \frac{(x+2)}{x}}{\frac{1}{x}} = \frac{2 - (x+2)}{2(x+2)} = \frac{-x}{2(x+2)} \cdot \frac{1}{x} = \frac{-1}{2(x+2)} = -1/4$$

$$(14) \lim_{x \rightarrow \infty} \frac{4x-3}{2-x} = \boxed{-4}$$

$$(15) \lim_{x \rightarrow 5} \begin{cases} x-4 & x \leq 5 \\ -x+3 & x > 5 \end{cases} \quad \begin{matrix} 5-4=1 \\ -5+3=-2 \end{matrix} \quad \boxed{\text{DNE}}$$

$$(16) \lim_{x \rightarrow \infty} \frac{7x^2-2}{(2x-1)(3-x)} = \boxed{\frac{7}{-2}}$$

$$(17) \lim_{x \rightarrow 2^+} \begin{cases} x^2-2 & x \leq 2 \\ 3x-5 & x > 2 \end{cases} \\ 3 \cdot 2 - 5 = \boxed{1}$$

$$(18) \lim_{x \rightarrow -\infty} \frac{3x^2-7x}{2x^3} = \boxed{0}$$

$$(19) \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{1}{2} \lim_{x \rightarrow 0} \frac{1-\cos x}{x} \\ 1 \cdot \frac{1}{2} \cdot 0 = \boxed{0}$$

$$(20) \lim_{x \rightarrow 1^-} \begin{cases} x^3 & x \leq 1 \\ -x+3 & x > 1 \end{cases} \\ 1^3 = \boxed{1}$$

$$(21) f(0) = 0^3 + 2(0) - 1 = -1 \\ f(1) = 1^3 + 2 - 1 = 2$$

(22) none @  $x=2$   
Removable  
discont

$$(23) f(x) = \begin{cases} x^2 & x < 1 \\ 2x-1 & 1 \leq x \leq 3 \\ -x+7 & x > 3 \end{cases}$$

$$1^2 = 2(1) - 1$$

$$2(3) - 1 \neq -3 + 7$$

jump/break

discont @  $x=3$

$$(24) f(x) = \frac{x^2 - (x-5)(x+2)}{(x+5)(x+5)}$$

RD  
none @

$$x=5$$

VA

$$x=-5$$

$$(25) 1^2 + 1 = c(1) + 4$$

$$2 = c + 4$$

$$\boxed{c = -2}$$