NATIONAL MATH + SCIENCE INITIATIVE

AP Calculus

Area Accumulation and Approximation

Presenter Notes

2016-2017 EDITION

Student Study Session - Presenter Notes

Thank you for agreeing to present at one of NMSI's Saturday Study Sessions. We are grateful you are sharing your time and expertise with our students. Saturday mornings can be a "tough sell" for students, so <u>we encourage you to incorporate strategies and techniques to encourage student movement and engagement</u>. Suggestions for different presentation options are included in this document. If you have any questions about the content or about presentation strategies, please contact Mathematics Director Charla Holzbog at cholzbog@nms.org or AP Calculus Content Specialist Karen Miksch at kmiksch@nms.org.

The material provided contains many released AP multiple choice and free response questions as well as some AP-like questions that we have created. The goal for the session is to let the students experience a variety of both types of questions to gain insight on how the topic will be presented on the AP exam. It is also beneficial for the students to hear a voice other than their teacher in order to help clarify their understanding of the concepts.

Suggestions for presenting:
The vast majority of the study sessions are on Saturday and students and teachers are coming to be WOWed! We want activities to engage the students as well as prepare them for the AP Exam. The following presenter notes include pacing suggestions (you only have 50 minutes!), solutions, and recommended engagement strategies.

Suggestions on how to prepare:

- The notes/summaries on the last page(s) are for reference. We want the students' time during the session to be focused on the questions as much as possible and not taking or reading the notes. As the questions are presented during the session, you may wish to refer the students back to those pages as needed. It is not our intent for the sessions to begin with a lecture over these pages.
- As you prepare, work through the questions in the packet noting the level of difficulty and topic or skill required for the questions.
- Design a plan for what questions you would like to cover with the group depending on their level of expertise. Some groups will be ready for the tougher questions while other groups will need more guidance and practice on the easier ones. Create an easy, medium, and hard listing of the questions prior to the session. This will allow you to adjust on the fly as you get to know the groups. In most instances, there will **not** be enough time to cover all the questions in the packet. Use your judgement on the amount of questions to cover based on the students' interactions. Remember to include both multiple choice and free response type questions. Discussions on test taking strategies and scoring of the free response questions are always great to include during the day.
- The concepts should have been previously taught; however, be prepared to "teach" the topic if you find out the students have not covered the concept prior in class. In sessions where multiple schools come together, you might have a mixture of students with and without prior knowledge on the topic. You will have to use your best judgement in this situation.
- Consider working through some free response questions before the multiple choice questions, or flipping back and forth between the two types of questions. Sometimes, if free response questions are saved for the last part of the session, it is possible students only get practice with one or two of them and most students need additional practice with free response questions.

- **Area Approximation and Accumulation Student Session-Presenter Notes**

This session includes a reference sheet at the back of the packet. We suggest that the presenter not spend time going over the reference sheet, but point it out to students so that they may refer to it if needed.

We suggest that students will work in small groups of 3 or 4 (depending on the size of the class)-arrange the desks prior to the start of the session.

We have intentionally included more material than can be covered in most Student Study Sessions to account for groups that are able to answer the questions at a faster rate. Use our presenter notes and your own judgment, based on the group of students, to determine the order and selection of questions to work in the session. Be sure to include a variety of types of questions (multiple choice, free response, calculator, and non-calculator) in the time allotted. Notice in the solutions guide the questions are categorized as 3, 4, or 5 indicating a typical question of the difficulty level (DL) for a student earning these qualifying scores on the AP exam.

I. 5 Minutes: Introductory Activity

- Arrange the students randomly into groups of 3 or 4. Before beginning the questions in the packet, display the table and question **on the following page** for the students. Ask students to work the question individually and then discuss their answer and method with their group while you actively walk around to monitor their progress and listen to their conversations. Use this time to gauge the level of knowledge of the group.
- After a few minutes, discuss the different methods to approximate the definite integral (right, left, or midpoint Riemann sum or trapezoid approximation) or allow students to come to the board to explain the method they used. Try not to spend more than 5 minutes on this activity.

II. 20 minutes: Free Response Practice

- Assign the free response questions, 11-14, to the students. Allow about 3 minutes for the students to work together on each question and allow about 2 minutes to discuss each question with the whole group. While the students are working each question, you can focus more attention on those students who struggled with the introductory activity.

III. 25 minutes: Multiple Choice Practice--*your choice of questions depending on the comfort level of the students during the introductory activity —some suggested questions are listed below.*

- Choose a few multiple choice questions to model for the students, and then choose several to assign to the groups. Selections should be based on the knowledge level of the group. Use a think-pair-share strategy for the assigned questions. Suggestion: use as 1, 5, 8, and 9 as models, then assign the remaining problems, 2, 3, 4, 6, 7, and 10, as time allows.

x	0	10	20	30	40
$f(x)$	5	9	11	15	20

The function f is continuous on the closed interval $[0, 40]$ and has values that are given in the table above. Using the data in the table, what is an approximation for $\int_0^{40} f(x)dx$?

Introductory Activity

Left Riemann Sum: $10(5+9+11+15)=400$

Right Riemann Sum: $10(9+11+15+20)=550$

Midpoint Riemann Sum: $20(9+15)=480$

Trapezoid Approximation:

$$\frac{1}{2}(5+9)(10)+\frac{1}{2}(9+11)(10)+\frac{1}{2}(11+15)(10)+\frac{1}{2}(15+20)(10)=70+100+130+175=475$$

Multiple Choice Answers

1. A (AP-like) DL: 4

$$\frac{1}{2}\left(4^1+4^{3/2}+4^2+4^{5/2}\right)=\frac{1}{2}(4+8+16+32)=\frac{1}{2}(60)=30$$

2. C 2008 AB10 DL: 3

Since $f(x)$ is decreasing, the Right Riemann Sum $<\int_1^3 f(x)\,dx<$ Left Riemann Sum.

Since $f(x)$ is decreasing and concave down, the Right Riemann Sum $<$ both the Midpoint Riemann Sum and the Trapezoidal Sum.

3. E 1988 BC18 (appropriate for AB) DL: 4

$$\frac{1}{2}(2)\left(\frac{e^4}{2}+2\frac{e^2}{2}+2\frac{e^0}{2}+\frac{e^{-2}}{2}\right)=\frac{1}{2}(e^4+2e^2+2e^0+e^{-2})$$

4. C 1998 AB85/BC85 DL: 3

$$\frac{1}{2}(3)(10+30)+\frac{1}{2}(2)(30+40)+\frac{1}{2}(1)(40+20)=160$$

5. D (AB Sample Question #8 from 2014 Fall AP Calculus Curriculum Framework) DL: 5

For this integral, a right Riemann sum with n terms is built from rectangles of width $\Delta x=\frac{5-3}{n}=\frac{2}{n}$. The height of each rectangle is determined by the function x^4 evaluated at the right endpoints of the n intervals. These endpoints are the $x-$values at $3+k\Delta x$, for k from 1 to n, since the integral starts at 3. The Riemann sum can be written as $\sum_{k=1}^{n}(3+k\Delta x)^4\Delta x$ where $\Delta x=\frac{2}{n}$, and its limit as $n\to\infty$ is $\int_3^5 x^4\,dx$.

6. B 2003 BC90 (appropriate for AB) DL: 3
   I. False. The area accumulated from 0 to 1 is greater than the area accumulated from 0 to 0.
   II. True. The area accumulated from 0 to 2 is greater than the area accumulated from 0 to 1.
   III. False. The area accumulated from 0 to 3 is greater than the area accumulated from 0 to 1.

7. A 2003 AB85/BC85 DL: 3
   Since using trapezoids give an over-approximation, the graph is concave up. Since right Riemann sums give an under-approximation, the graph is decreasing. The only graph that is concave up and decreasing is choice (A).

8. E 1998 BC91 (appropriate for AB) DL: 3
   $$11\frac{\text{ft}}{\text{sec}}+2\text{ sec}\left(5\frac{\text{ft}}{\text{sec}^2}+2\frac{\text{ft}}{\text{sec}^2}+8\frac{\text{ft}}{\text{sec}^2}\right)=41\frac{\text{ft}}{\text{sec}}$$

9. B (AP-like) DL: 4
   Since f is an increasing function, a left Riemann sum gives an under-approximation and a right Riemann sum gives an over-approximation. Therefore,
   $$\text{Left Riemann Sum} < \int_0^2 f(x)\,dx < \text{Right Riemann Sum}$$
   $$36 < \int_0^2 f(x)\,dx < 50$$

10. D (AP-like) DL: 3

$$35\text{ gallons}+3\text{ min}\left(5.1\,\frac{\text{gallons}}{\text{min}}\right)+4\text{ min}\left(4.7\,\frac{\text{gallons}}{\text{min}}\right)+2\text{ min}\left(4.4\,\frac{\text{gallons}}{\text{min}}\right)=77.9\text{ gallons}$$

Free Response Solutions

11. 2007 AB5/BC5

(c) $\int_0^{12} r'(t)dt \approx$ $2(4.0)+3(2.0)+2(1.2)+4(0.6)+1(0.5)=19.3\text{ ft}$ $\int_0^{12} r'(t)dt$ is the change in the radius, in feet, from $t=0$ to $t=12$ minutes.	2 { 1: approximation 1: explanation
(d) Since r is concave down, r' is decreasing on $0<t<12$. Therefore, this approximation, 19.3 ft, is less then $\int_0^{12} r'(t)dt$.	1: conclusion with reason
Unit of ft^3/min in part (b) or ft in part (c)	1: units in (b) or (c)

12. 2011B AB5/BC5

(b) $\int_0^{60} \lvert v(t) \rvert \, dt$ is the total distance, in meters, that Ben rides over the 60-second interval $t = 0$ to $t = 60$. $\int_0^{60} \lvert v(t) \rvert \, dt \approx 2.0 \cdot 10 + 2.3(40-10) + 2.5(60-40) = 139$ meters	2 { 1: meaning of integral 1: approximation 1: units in (a) or (b)

13. 2014 AB4/BC4

(c) $s_A(12) = s_A(2) + \int_2^{12} v_A(t)\,dt = 300 + \int_2^{12} v_A(t)\,dt$ $\int_2^{12} v_A(t)\,dt \approx 3 \bullet \frac{100+40}{2} + 3 \bullet \frac{40-120}{2} + 4 \bullet \frac{-120-150}{2}$ $= -450$ $s_A(12) \approx 300 - 450 = -150$ The position of Train A at time $t = 12$ minutes is approximately 150 meters west of Origin Station.	3 { 1: position expression 1: trapezoidal sum 1: position at time $t = 12$

14. 2013 AB3

(a) $\frac{1}{6}\int_0^6 C(t)dt \approx \frac{1}{6}\left[2 \bullet C(1) + 2 \bullet C(3) + 2 \bullet C(5)\right]$ $= \frac{1}{6}(2 \bullet 5.3 + 2 \bullet 11.2 + 2 \bullet 13.8)$ $= \frac{1}{6}(60.6) = 10.1$ ounces $\frac{1}{6}\int_0^6 C(t)dt$ is the average amount of coffee in the cup, in ounces, over the time interval $0 \le t \le 6$ minutes.	3: { 1: midpoint sum 1: approximation 1: interpretation

