AP Calculus

Derivatives and Their Applications

Presenter Notes

2017-2018 EDITION

Student Study Session - Presenter Notes

Thank you for agreeing to present at one of NMSI's Saturday Study Sessions. We are grateful you are sharing your time and expertise with our students. Saturday mornings can be a "tough sell" for students, so we encourage you to incorporate strategies and techniques to encourage student movement and engagement. Suggestions for different presentation options are included in this document. If you have any questions about the content or about presentation strategies, please contact Mathematics Director Charla Holzbog at cholzbog@nms.org or AP Calculus Content Specialist Karen Miksch at kmiksch@nms.org.

The material provided contains many released AP multiple choice and free response questions as well as some AP-like questions that we have created. The goal for the session is to let the students experience a variety of both types of questions to gain insight on how the topic will be presented on the AP exam. It is also beneficial for the students to hear a voice other than their teacher in order to help clarify their understanding of the concepts.

Suggestions for presenting:
The vast majority of the study sessions are on Saturday and students and teachers are coming to be WOWed! We want activities to engage the students as well as prepare them for the AP Exam. The following presenter notes include pacing suggestions (you only have 50 minutes!), solutions, and recommended engagement strategies.

Suggestions on how to prepare:

- The notes/summaries on the last page(s) are for reference. We want the students' time during the session to be focused on the questions as much as possible and not taking or reading the notes. As the questions are presented during the session, you may wish to refer the students back to those pages as needed. It is not our intent for the sessions to begin with a lecture over these pages.
- As you prepare, work through the questions in the packet noting the level of difficulty and topic or skill required for the questions.
- Design a plan for what questions you would like to cover with the group depending on their level of expertise. Some groups will be ready for the tougher questions while other groups will need more guidance and practice on the easier ones. Create an easy, medium, and hard listing of the questions prior to the session. This will allow you to adjust on the fly as you get to know the groups. In most instances, there will **not** be enough time to cover all the questions in the packet. Use your judgement on the amount of questions to cover based on the students' interactions. Remember to include both multiple choice and free response type questions. Discussions on test taking strategies and scoring of the free response questions are always great to include during the day.
- The concepts should have been previously taught; however, be prepared to "teach" the topic if you find out the students have not covered the concept prior in class. In sessions where multiple schools come together, you might have a mixture of students with and without prior knowledge on the topic. You will have to use your best judgement in this situation.
- Consider working through some free response questions before the multiple choice questions, or flipping back and forth between the two types of questions. Sometimes, if free response questions are saved for the last part of the session, it is possible students only get practice with one or two of them and most students need additional practice with free response questions.

Derivatives and Their Applications Student Session-Presenter Notes

This session includes a reference sheet at the back of the packet. We suggest that the presenter spends few minutes only on the differentiation rules and does not spend time going over the whole reference sheet, but may point it out to students that it is available to refer to if needed.

We suggest that students will work in pairs (depending on the size of the class)- arrange the desks prior to the start of the session.

We have intentionally included more material than can be covered in most Student Study Sessions (this one is especially long) to account for groups that are able to answer the questions at a faster rate. Use our presenter notes and your own judgment, based on the group of students, to determine the order and selection of questions to work in the session. Be sure to include a variety of types of questions (multiple choice, free response, calculator, and non-calculator) in the time allotted. Notice in the solutions guide the questions are categorized as 3, 4, or 5 indicating a typical question of the difficulty level (DL) for a student earning these qualifying scores on the AP exam.

I. 5 Minute Introductory Activity

- When students enter the room, ask them to complete the reference sheet about the derivatives as they wait a few minutes for the start of the session.
- When the session starts, display the answers and let the students first mark and then correct any mistakes.
- Ask students to take the derivatives of $\frac{d}{dx}\left(\sqrt{u}\right) = ?$ and $\frac{d}{dx}\left(\frac{1}{u}\right) = ?$ using power and chain rules. Remind them these will appear in the AP exam more often than other radical and rational functions, and it is beneficial to memorize these two rules as well.
- Let the students to work in their pairs on problems #1, 3, 5 and 6. Please walk around to actively monitor them and also determine the level of the group. Check the answers.

II. 15 minutes Differentiation Rules Practice

- Model numeric differentiation using questions FR 30 and FR 31. Compare and contrast these two questions. Remind students that when estimating the derivative using the given numeric data, they need to use the closest possible data points (question 31 has three acceptable answers), must show the difference quotient, use ~ symbol, in non-calculator questions they don't need to reduce/simplify their answers and in most cases must indicate the units and explain the meaning of their answers.
- Let the students to work on problems #15, 16, 20, 25, 34 and 37 in pairs in a speed-dating activity: **Speed Dating:** (or inner/outer circle) Assign half of the students to a particular station (A,B,C, etc.). The other students are assigned to begin the speed-dating activity at a specific station and then will rotate through the other stations after an allotted amount of time for a multiple choice (presenter calls out the question to be

worked) or part of a free response question of your choice. Discuss and grade each question as the game proceeds, or wait until all the "dates" are completed to discuss and correct.

- Extra practice questions on differentiation rules: #2, 4, 7, 9, 11, 12, 14, 24, 27, 29, 35. - Use more of these as time allows during the speed-dating activity or have them work in pairs on some extra problems in class or advise the students to work on these problems on their own time when they use this packet to study for their AP exam.

III. 10 minutes Tangent Line Approximation Practice

- Remind students that *tangent line approximation* may appear in the AP exam under different names as *linearization* or *local linear approximation*.
- Model problems MC #13, 18, 21 and FR 32.
- Extra practice questions on tangent line approximation: #17, 36

IV. 10 minutes Implicit Differentiation Practice

- Model problem #26
- Let the students work in their group on the free response problem #28. After few minutes, display the rubric and let the students score their answers.
- Extra practice questions on implicit differentiation: #8, 10.

V. 10 minutes Related Rates Practice

- Model problem #22
- Let the students work in their group on the free response problem #33. After few minutes, display the rubric and let the students score their answers.
- Extra practice question on related rates: #19.
- **Update Option**: Please notice the QR Code activity to accompany this study session that was created by Bryan Passwater. Note that it includes some of the questions in Activity B that go along with the latter part of the session and contains more general questions from the packet as well as a few additional problems. Feel free to use these as an engaging, quick culminating activity for your students-instructions are included in the file.

We know this is a long packet and want to encourage our presenters to "pick and choose" the best questions for the group of students in your session. Encourage the students and teachers present to be sure to revisit the questions that you didn't have time to cover in the classroom.

Derivatives and Their Applications Solutions

Multiple Choice:

1. A (1993 AB24) DL: 4

$$f(x)=\left(x^2-2x-1\right)^{\frac{2}{3}} \to f'(x)=\frac{2}{3}\left(x^2-2x-1\right)^{\frac{-1}{3}}(2x-2)=\frac{2(2x-2)}{3\left(x^2-2x-1\right)^{\frac{1}{3}}}$$

$$f'(0)=\frac{-4}{-3}=\frac{4}{3}$$

2. E (1985 AB2) DL: 4

$$f'(x)=4(2x+1)^3(2)=8(2x+1)^3$$

$$f''(x)=24(2x+1)^2(2)=48(2x+1)^2$$

$$f'''(x)=96(2x+1)^1(2)=192(2x+1)$$

$$f^{(4)}(x)=384$$

3. A (AP-like) DL: 4

$$h'(x)=f(x)g'(x)+g(x)f'(x)+\frac{g(x)f'(x)-f(x)g'(x)}{\left[g(x)\right]^2}$$

$$h'(5)=(3)(5)+(-2)(4)+\frac{(-2)(4)-(3)(5)}{(-2)^2}=7-\frac{23}{4}=\frac{5}{4}$$

4. A (AP-like) DL: 3

$$h'(x)=f'\left(g(2x)\right)g'(2x)(2)$$

$$h'(2)=f'\left(g(4)\right)g'(4)(2)=f'(3)g'(4)(2)=(-5)(9)(2)=-90$$

5. C (AP-like) DL: 3

$$h'(x)=2g(x)g'(x)$$

$$h'(3)=2g(3)g'(3)=2(6)(2)=24$$

6. E (1988 AB18) DL: 3

$$\frac{dy}{dx}=-2\sin\left(\frac{x}{2}\right)\left(\frac{1}{2}\right)=-\sin\left(\frac{x}{2}\right); \quad \frac{d^2y}{dx^2}=-\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\right)=\frac{-1}{2}\cos\left(\frac{x}{2}\right)$$

7. D (AB Sample Question #7 from AP Calculus Course and Exam Description) DL: 4

8. B (2003 AB26) DL: 3

$3y^2 - 2x^2 + 2xy = 6$ (moving $-2xy$ from right to the left side of the equation will create a positive coefficient in front of the term that requires product rule and will avoid some potential errors)

$$6y\frac{dy}{dx} - 4x + 2x\frac{dy}{dx} + y(2) = 0$$

$$\frac{dy}{dx}(6y + 2x) = 4x - 2y$$

$$\frac{dy}{dx} = \frac{4x - 2y}{6y + 2x} = \frac{4(3) - 2(2)}{6(2) + 2(3)} = \frac{8}{18} = \frac{4}{9}$$

9. B (AB Sample Question #18 from AP Calculus Course and Exam Description) DL: 3

Instantaneous rate of change is another name for the derivative. Students should use their calculators to evaluate the derivative of the given function at $t = 90$ days.

10. D (2008 AB16) DL: 4

$$\cos(xy)\left[x\frac{dy}{dx} + y(1)\right] = 1$$

$$x\frac{dy}{dx} + y = \frac{1}{\cos(xy)} \rightarrow x\frac{dy}{dx} = \frac{1}{\cos(xy)} - y = \frac{1 - y\cos(xy)}{\cos(xy)} \rightarrow \frac{dy}{dx} = \frac{1 - y\cos(xy)}{x\cos(xy)}$$

11. A (1995 BC5, appropriate for AB) DL: 4

$$h'(x) = f'(g(x))g'(x)$$

$h''(x)$ requires the use of the product rule,

$$h''(x) = f'(g(x))g''(x) + g'(x)f''(g(x))g'(x)$$

$$h''(x) = f'(g(x))g''(x) + (g'(x))^2 f''(g(x))$$

12. A (AP-like) DL: 3

$$f'(x) = \frac{1}{1 + x^2}$$

$$f'(\sqrt{3}) = \frac{1}{1 + (\sqrt{3})^2} = \frac{1}{4}$$

13. C (AP-like) DL: 4

$$f(x)=\sqrt{e^x+3}\rightarrow f(0)=2$$

$$f'(x)=\frac{e^x}{2\sqrt{e^x+3}}\rightarrow f'(0)=\frac{1}{4}$$

Tangent line equation : $y=\frac{1}{4}(x-0)+2$, when $x=\frac{1}{2}$, $y=\frac{1}{8}+2=2.125$

14. A (AP-like) DL: 3

$$f'(x)=\frac{1-3e^{-3x}}{x+4+e^{-3x}} \quad \rightarrow \quad f'(0)=\frac{1-3e^0}{0+4+e^0}=\frac{-2}{5}$$

15. D (2008 AB3) DL: 4

$$f'(x)=(x-1)3\left(x^2+2\right)^2(2x)+\left(x^2+2\right)^3(1)$$

factoring out the common term $\left(x^2+2\right)^2$ helps to condense the answer;

$$f'(x)=\left(x^2+2\right)^2\left[(x-1)6x+\left(x^2+2\right)\right]$$

$$f'(x)=\left(x^2+2\right)^2\left[6x^2-6x+x^2+2\right]=\left(x^2+2\right)^2\left[7x^2-6x+2\right]$$

16. A (2008 AB28) DL: 4

If $f(x)$ contains (a,b) then its inverse $g(x)=f^{-1}(x)$ will contain (b,a). Their derivatives are related with the equation $g'(b)=\frac{1}{f'(a)}$.

Similarly, when $f(x)$ contains $(6,3)$ then its inverse $g(x)=f^{-1}(x)$ contains $(3,6)$.

Then $g'(3)=\frac{1}{f'(6)}=\frac{1}{-2}$.

17. D (AP-like) DL: 4

slope of the secant line ; $m = \dfrac{15-3}{4-1} = 4$

$h'(x) = 3x^2 + 2kx$

$h'(-2) = 12 - 4k$

setting $h'(-2) = 4$ and solving for k yields $k = 2$

18. D (AB Sample Question #5 from AP Calculus Course and Exam Description) DL: 4

The function $g(x)$ and its tangent line will contain the same point (point of tangency), therefore $g\left(\dfrac{1}{2}\right) = y\left(\dfrac{1}{2}\right) = 4\left(\dfrac{1}{2}\right) + 1 = 3$

At the point of tangency, they also have the same slope, therefore $g'\left(\dfrac{1}{2}\right) = y'\left(\dfrac{1}{2}\right) = 4$

$g\left(\dfrac{1}{2}\right) + g'\left(\dfrac{1}{2}\right) = 4 + 3 = 7$

19. A (AB Sample Question #4 from the early version of AP Calculus Course and Exam Description) DL: 4

$\dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt} \quad \rightarrow \quad -2\pi = 4\pi(5)^2 \dfrac{dr}{dt} \quad \rightarrow \quad \dfrac{dr}{dt} = \dfrac{-2}{4(25)} = \dfrac{-1}{50}$

$\dfrac{dA}{dt} = 8\pi r \dfrac{dr}{dt} = 8\pi(5)\dfrac{-1}{50} = \dfrac{-4\pi}{5}$

20 . B (AP-like) DL: 4

$f(x) = e^{\sin x} - 1 = 0 \quad \rightarrow \quad e^{\sin x} = 1 \quad \rightarrow \quad \sin(x) = \ln 1 = 0 \quad \rightarrow \quad x = \pi$ is the only solution on [1,4]

$f'(x) = e^{\sin x} \cos x$

$f'(\pi) = e^{\sin \pi} \cos \pi = e^0(-1) = -1$

21 . A (2008 AB18) DL: 5

The tangent line, $y = -x + k$, and the graph of the function y has the same slope, -1, at the point of tangency;

$y' = 2x + 3 = -1$ at $x = -2$

At the point of tangency, they also have the same y-coordinate;

$4 - 6 + 1 = 2 + k$

$k = -3$

22 . D (1998 AB90) DL: 4

$$A = \frac{1}{2}bh$$

$$\frac{dA}{dt} = \frac{1}{2}\left(b\frac{dh}{dt} + h\frac{db}{dt}\right)$$

$$\frac{dA}{dt} = \frac{1}{2}\left(b(-3) + h(3)\right) = \frac{3}{2}(-b + h)$$

When $b > h$, $\frac{dA}{dt}$ is negative therefore A decreases.

23. A (2008 AB11) DL: 3

$$f'(x) = x3(1-2x)^2(-2) + (1-2x)^3$$

$$f'(1) = -6 + (-1) = -7$$

24. B (AB Sample Question #3 from AP Calculus Course and Exam Description) DL: 4

$$f'(x) = \cos(\ln(2x))\frac{1}{2x}(2)$$

$$f'(x) = \frac{\cos(\ln(2x))}{x}$$

25. D (AP-like) DL: 4

$A = 2xy = 2x\left(9 - x^2\right) = 18x - 2x^3$

$\frac{dA}{dx} = 18 - 6x^2 = 0$ and changes sign from positive to negative at $x = \sqrt{3}$.

The domain is x : $(0, 3)$, at $x = \sqrt{3}$, there must be the absolute max of the area function.

$A\big|_{x=\sqrt{3}} = 2\sqrt{3}\left(9 - \left(\sqrt{3}\right)^2\right) = 12\sqrt{3}$

26. B (AP-like) DL: 4

$\frac{d^2y}{dx^2} = 2y\frac{dy}{dx} + \frac{dy}{dx}$

at $x = 3$ and $y = 1$, $\frac{dy}{dx} = 1^2 + 1 = 2$

then, $\frac{d^2y}{dx^2} = 2(1)(2) + (2) = 6$

Free Response

27. 2003-Form B- AB6a

(a) $f''(x) = \sqrt{f(x)} + x \cdot \frac{f'(x)}{2\sqrt{f(x)}} = \sqrt{f(x)} + \frac{x^2}{2}$

$f''(3) = \sqrt{25} + \frac{9}{2} = \frac{19}{2}$

3 : { 2 : $f''(x)$
< −2 > product or chain rule error
1 : value at $x = 3$ }

28. 2000 AB/BC 5

(a) $y^2 + 2xy\dfrac{dy}{dx} - 3x^2y - x^3\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}\left(2xy - x^3\right) = 3x^2y - y^2$

$\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$

$2\begin{cases} 1: \text{implicit differentiation} \\ 1: \text{verifies expression for } \dfrac{dy}{dx} \end{cases}$

(b) When $x = 1$, $y^2 - y = 6$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3,\ y = -2$

At $(1,3)$, $\dfrac{dy}{dx} = \dfrac{9-9}{6-1} = 0$

Tangent line equation is $y = 3$

At $(1,-2)$, $\dfrac{dy}{dx} = \dfrac{-6-4}{-4-1} = \dfrac{-10}{-5} = 2$

Tangent line equation is $y + 2 = 2(x-1)$

$4\begin{cases} 1: \ y^2 - y = 6 \\ 1: \ \text{solves for } y \\ 2: \ \text{tangent lines} \end{cases}$

Note: 0/4 if not solving an equation of the form $y^2 - y = k$

(c) Tangent line is vertical when $2xy - x^3 = 0$

$x\left(2y - x^2\right) = 0$ gives $x = 0$ or $y = \dfrac{1}{2}x^2$

There is no point on the curve with x-coordinate 0.

When $y = \dfrac{1}{2}x^2$, $\dfrac{1}{4}x^5 - \dfrac{1}{2}x^5 = 6$

$-\dfrac{1}{4}x^5 = 6$

$x = \sqrt[5]{-24}$

$3\begin{cases} 1: \ \text{sets denominator of } \dfrac{dy}{dx} \text{ equal to } 0 \\ 1: \ \text{substitutes } y = \dfrac{1}{2}x^2 \text{ or } x = \pm\sqrt{2y} \\ \quad\ \text{into the equation for the curve} \\ 1: \ \text{solves for } x\text{-coordinate} \end{cases}$

29. 2014 AB/BC 1a,b

(a) $\dfrac{A(30)-A(0)}{30-0} = -0.197(\text{or } -0.196)\text{lbs/day}$	1: answer with units
(b) $A'(15) = -0.164(\text{or } -0.163)$ The amount of grass clippings in the bin is decreasing at a rate of 0.164 (or 0.163) lbs/day at time $t = 15$ days.	$2\begin{cases}1: & A'(15)\\ 1: & \text{interpretation}\end{cases}$

30. 2012 AB/BC 1a

(a) $W'(12) \approx \dfrac{W(15)-W(9)}{15-9} = \dfrac{67.9-61.8}{6}$ $= 1.017$ (or 1.016) The water temperature is increasing at a rate of approximately 1.017 °F per minute at time $t = 12$ minutes.	$2:\begin{cases}1:\text{estimate}\\ 1:\text{interpretation with units}\end{cases}$

31. 2001 AB/BC 2a

(a) Difference quotient; e.g. $W'(12) \approx \dfrac{W(15)-W(12)}{15-12} = -\dfrac{1}{3}$ °C/day or $W'(12) \approx \dfrac{W(12)-W(9)}{12-9} = -\dfrac{2}{3}$ °C/day or $W'(12) \approx \dfrac{W(15)-W(9)}{15-9} = -\dfrac{1}{2}$ °C/day	$2:\begin{cases}1:\text{difference quotient}\\ 1:\text{answer (with units)}\end{cases}$

32. 2010 AB6a,b

(a) $f'(1) = \left.\frac{dy}{dx}\right\|_{(1,\,2)} = 8$ An equation of the tangent line is $y = 2 + 8(x-1)$.	$2\begin{cases} 1: & f'(1) \\ 1: & \text{answer} \end{cases}$
(b) $f(1.1) \approx 2.8$ Since $y = f(x) > 0$ on the interval $1 \le x \le 1.1$, $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right) > 0$ on this interval. Therefore on the interval $1 < x < 1.1$, the line tangent to the graph of $y = f(x)$ at $x = 1$ lies below the curve and the approximation 2.8 is less than $f(1.1)$.	$2\begin{cases} 1: & \text{approximation} \\ 1: & \text{conclusion with explanation} \end{cases}$

33. 2002-Form B-AB6

(a) Distance $= \sqrt{3^2+4^2} = 5$ km	1 : answer
(b) $r^2 = x^2 + y^2$ $2r\dfrac{dr}{dt} = 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt}$ or explicitly: $r = \sqrt{x^2+y^2}$ $\dfrac{dr}{dt} = \dfrac{1}{2\sqrt{x^2+y^2}}\left(2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt}\right)$ At $x = 4$, $y = 3$, $\dfrac{dr}{dt} = \dfrac{4(-15)+3(10)}{5} = -6$ km/hr	4 { 1 : expression for distance 2 : differentiation with respect to t $< -2 >$ chain rule error 1 : evaluation
(c) $\tan\theta = \dfrac{y}{x}$ $\sec^2\theta\dfrac{d\theta}{dt} = \dfrac{\frac{dy}{dt}x - \frac{dx}{dt}y}{x^2}$ At $x = 4$ and $y = 3$, $\sec\theta = \dfrac{5}{4}$ $\dfrac{d\theta}{dt} = \dfrac{16}{25}\left(\dfrac{10(4)-(-15)(3)}{16}\right)$ $= \dfrac{85}{25} = \dfrac{17}{5}$ radians/hr	4 { 1 : expression for θ in terms of x and y 2 : differentiation with respect to t $< -2 >$ chain rule, quotient rule, or transcendental function error note: 0/2 if no trig or inverse trig function 1 : evaluation

34. 2014 AB/BC 3d

(d) $p'(x) = f'(x^2 - x)(2x-1)$ $p'(-1) = f'(2)(-3) = (-2)(-3) = 6$	3 { 2: $p'(x)$ 1: answers

35. 2011-Form B- AB 2c

(c) $r'(3) = 50$ The rate at which water is draining out of the tank at time $t = 3$ hours is increasing at 50 liters/hour^2.	2 { 1: $r'(3)$ 1: meaning of $r'(3)$

36. 2009-FormB- AB5a

(a) $g(1) = e^{f(1)} = e^2$ $g'(x) = e^{f(x)} f'(x)$, $g'(1) = e^{f(1)} f'(1) = -4e^2$ The tangent line is given by $y = e^2 - 4e^2(x-1)$.	$3: \begin{cases} 1: g'(x) \\ 1: g(1) \text{ and } g'(1) \\ 1: \text{tangent line equation} \end{cases}$

37. AP-like

$h'(0)$ does not exist.

Even if $\lim_{x\to 0^-} h'(x) = \lim_{x\to 0^-} 4x^3 + 3 = 3$ and $\lim_{x\to 0^+} h'(x) = \lim_{x\to 0^-} 3e^{3x} = 3$, since $\lim_{x\to 0^-} h(x) \neq \lim_{x\to 0^+} h(x)$, $h(x)$ is not continuous at $x = 0$, therefore $h(x)$ is not differentiable at $x = 0$.

General Rules

Definition of Derivative:

$$\frac{d}{dx}\left(f(x)\right)= f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Sum and Difference Rule:

$$\frac{d}{dx}\left(f(x)\pm g(x)\right)= f'(x)\pm g'(x)$$

Constant Multiple Rule:

$$\frac{d}{dx}\left(k\cdot f(x)\right)= k\cdot f'(x)$$

Product Rule:

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right)= f'(x)g(x)+f(x)g'(x)$$

Quotient Rule:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)= \frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$$

Chain Rule:

$$\frac{d}{dx}\left(f(g(x))\right)= f'(g(x))\cdot g'(x)$$

Particular Rules

Power Rule:

$$\frac{d}{dx}\left(x^n\right)= nx^{n-1}$$

Exponential functions:

$$\frac{d}{dx}\left(e^u\right)= e^u\cdot\frac{du}{dx}$$

Bases Other than e: $\frac{d}{dx}(a^u) = a^u(\ln a)\frac{du}{dx}$

Trigonometric Functions:

$$\frac{d}{dx}\left(\sin(u)\right)=\cos(u)\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left(\cos(u)\right)=-\sin(u)\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left(\tan(u)\right)=\sec^2(u)\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left(\csc(u)\right)=-\csc(u)\cot(u)\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left(\sec(u)\right)=\sec(u)\tan(u)\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left(\cot(u)\right)=-\csc^2(u)\cdot\frac{du}{dx}$$

Logarithmic Functions:

$$\frac{d}{dx}\left(\ln(u)\right)= \frac{1}{u}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left(\log_a(u)\right)=\frac{d}{dx}\left(\frac{\ln u}{\ln a}\right)=\frac{1}{\ln a}\cdot\frac{1}{u}\cdot\frac{du}{dx}$$

Inverse Trigonometric Functions:

$$\frac{d}{dx}\left(\sin^{-1}(u)\right)=\frac{1}{\sqrt{1-u^2}}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left(\tan^{-1}(u)\right)=\frac{1}{1+u^2}\cdot\frac{du}{dx}$$