**8.8 Improper Integrals**

|  |
| --- |
| Integrals such as  are called **improper integrals**.  They are evaluated by rewriting the integral as a proper integral and then using limits:  , and |

Ex. 

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Ex. 

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Ex. 

|  |
| --- |
| **Homework**:  P. 585: 15,17,33,45 |

**9.1-9.2 Sequences, Series, and the Nth Term Test**

**\*\*\*\* Note I mention a sub in the video. Ignore that please!!!**

Sequence  =

converges if:

diverges if:

Series

2 Big Questions

1)

2)

Ex. 

Ex. 

To “Stand a Chance” to converge:

Ex. 

Ex. 

Ex. 

NEVER use the Nth term test to

|  |
| --- |
| **Homework**:  Worksheet |

**9.2 Geometric Series and Telescopic Series**

What we know so far:

Ex. 

Ex. 

Ex. 

=

Converges if:

Converges to:

Ex.

Ex. 

Ex. 

Ex.

Ex. 0.999999999……..

|  |
| --- |
| **Homework**:  HW - P.612: 7,9,11,23,25, 59,61,63,79 |

**9.3 Integral Test and *p*-Series**

|  |
| --- |
| **Integral Test**  If *f* is positive, continuous, and decreasing for  and  either both converge or both diverge. |

Ex. Determine whether the following series converge or diverge.

(a) 

(b) 

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |
| --- |
| **Integral Test Remainder:**  If *f* is positive, continuous, and decreasing for  and  both converge, then the series converges to *S*, and the remainder,  is bounded  by . |

Ex. Approximate the sum of the convergent series  by using six terms. Include an

estimate of the maximum error for your approximation.

|  |
| --- |
| ***p*-Series**  A series of the form  is  called a *p*-series, where *p* is a positive constant.  For *p* = 1, the series  is called the harmonic series. |

|  |
| --- |
| ***p*-Series Test**  The *p*-series  a)  b)  c) |

Ex. 

|  |
| --- |
| **Homework:** P. 620: 1, 35, 36, 61, 79, 81, 83 |

**9.4 Comparison of Series**

|  |
| --- |
| **Direct Comparison Test**  If  1) If  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.  2) If  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. |

Ex. Determine whether the following converge or diverge.

(a) 

(b) 

(c) 

|  |
| --- |
| **Limit Comparison Test**  Suppose where *L* is both finite and positive.  Then the two series  either both converge or both diverge. |

Ex. Determine whether the following converge or diverge.

(a) 

(b) 

|  |
| --- |
| **Homework**: P. 628: 3, 5, 7, 15, 19, 23, 29-36 all |

**9.5** **Alternating Series**

An alternating series is a series whose terms are alternately positive and negative.

Examples: 



In general, just knowing that  tells us very little about the convergence of the series ; however, it turns out that an alternating series must converge if the terms have a limit of 0 and the terms decrease in magnitude.

|  |
| --- |
| **Alternating Series Test (Also known as Leibniz Test for Alternating Series)**  Let  The alternating series  converge if the following two conditions are met:  1)  and 2)  for all *n*. |
| In other words, a series converges if its terms:  1) alternate in sign;  2) decrease in magnitude; and  3) have a limit of 0. |

**Note**: This does **not** say that if , the series diverges by the Alternating Series

Test. The Alternating Series Test can **ONLY** be used to prove **convergence**.

If , then the series diverges by the ***n*th Term Test for Divergence**,

not by the Alternating Series Test.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Ex. Determine whether the following series converge or diverge.

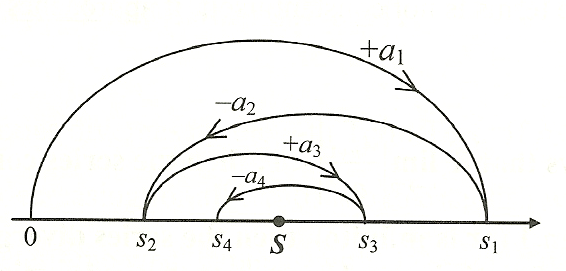
(a) 

(b) 

(c) 

This series is called the **alternating harmonic series**.

If an alternating series converges to a sum *S*, then its partial sums jump around *S* from side to side with decreasing distances from *S.* The figure below shows the first four partial sums of an alternating series which converges to a sum *S*.



\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Definitions:**

 is **absolutely convergent** if  converges.

 is **conditionally convergent** if converges but diverges.

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Ex. Determine whether the given alternating series converges or diverges. If it converges, determine whether it is absolutely convergent or conditionally convergent.



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| **Homework:** P. 637: 11, 19, 25, 28, 51, 57 |

**9.5 Alternating Series, Day 2**

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| --- |
| **Alternating Series Remainder**  If a series has terms that are alternating, decreasing in magnitude, and having a limit of 0, then the series converges so that it has a sum *S*. If the sum *S* is approximated by the nth partial sum, , then the error in the approximation, , will be less than the absolute value of the first omitted or truncated term. |
| In other words, if the three conditions are met, you can approximate the sum of the series by using the *n*th partial sum, , and your error will be bounded by the absolute value of the first truncated term. |

Ex. Given the series .

(a) Approximate the sum, *S*, of the series by using its first four terms.

(b) Explain why the estimate found in (a) differs from the actual value by less than 

(c) Use your results to explain why 

Ex. How many terms are needed to approximate the sum of the series  so that the

estimate differs from the actual sum by less than  Justify your answer.

|  |
| --- |
| **Homework:** Worksheet |

**9.6 Ratio Test**

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| --- |
| **Ratio Test**  Let  be a series of nonzero terms.  1)  converges if .  2)  diverges if .  3) If , the Ratio Test is inconclusive so another test would need to be used. |

Ex. Determine whether the following converge or diverge.

(a) 

(b) 

(c) 

|  |
| --- |
| **Root Test**  1)  converges if .  2)  diverges if .  3) If , the Root Test is inconclusive so another test would  need to be used. |

Ex. Determine whether the following converge or diverge.

(a) 

(b) 

|  |
| --- |
| **Homework:** P. 645: 13, 15, 19, 23, 25, 33, 37 |